General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MFP3

Unit Further Pure 3

Monday 10 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = (x - y)\sqrt{x + y}$$

and $$y(2) = 1$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.2$, to obtain an approximation to $y(2.2)$, giving your answer to three decimal places. *(5 marks)*

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2 The Cartesian equation of a circle is $(x + 8)^2 + (y - 6)^2 = 100$.

Using the origin O as the pole and the positive x-axis as the initial line, find the polar equation of this circle, giving your answer in the form $r = p\sin\theta + q\cos\theta$. *(4 marks)*

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3 (a) Find the values of the constants a, b and c for which $a + bx + cx\mathrm{e}^{-3x}$ is a particular integral of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 3x - 8\mathrm{e}^{-3x}$$ *(5 marks)*

(b) Hence find the general solution of this differential equation. *(3 marks)*

(c) Hence express y in terms of x, given that $y = 1$ when $x = 0$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} \to -1$ as $x \to \infty$. *(4 marks)*

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4 Evaluate the improper integral

$$\int_0^{\infty} \left(\frac{2x}{x^2 + 4} - \frac{4}{2x + 3} \right) \mathrm{d}x$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant. *(6 marks)*

5 (a) Differentiate $\ln(\ln x)$ with respect to x. *(1 mark)*

(b) (i) Show that $\ln x$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{1}{x \ln x} y = 9x^2, \quad x > 1$$ *(2 marks)*

(ii) Hence find the solution of this differential equation, given that $y = 4e^3$ when $x = e$. *(6 marks)*

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6 It is given that $y = (4 + \sin x)^{\frac{1}{2}}$.

(a) Express $y\frac{dy}{dx}$ in terms of $\cos x$. *(2 marks)*

(b) Find the value of $\frac{d^3y}{dx^3}$ when $x = 0$. *(5 marks)*

(c) Hence, by using Maclaurin's theorem, find the first four terms in the expansion, in ascending powers of x, of $(4 + \sin x)^{\frac{1}{2}}$. *(2 marks)*

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7 A differential equation is given by

$$\sin^2 x \frac{d^2y}{dx^2} - 2 \sin x \cos x \frac{dy}{dx} + 2y = 2 \sin^4 x \cos x, \quad 0 < x < \pi$$

(a) Show that the substitution

$$y = u \sin x$$

where u is a function of x, transforms this differential equation into

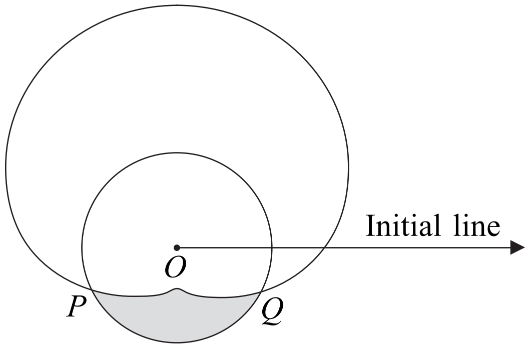
$$\frac{d^2u}{dx^2} + u = \sin 2x$$ *(5 marks)*

(b) Hence find the general solution of the differential equation

$$\sin^2 x \frac{d^2y}{dx^2} - 2 \sin x \cos x \frac{dy}{dx} + 2y = 2 \sin^4 x \cos x$$

giving your answer in the form $y = f(x)$. *(6 marks)*

8 The diagram shows a sketch of a curve and a circle.



The polar equation of the curve is

$$r = 3 + 2\sin\theta, \quad 0 \leqslant \theta \leqslant 2\pi$$

The circle, whose polar equation is $r = 2$, intersects the curve at the points P and Q, as shown in the diagram.

(a) Find the polar coordinates of P and the polar coordinates of Q. *(3 marks)*

(b) A straight line, drawn from the point P through the pole O, intersects the curve again at the point A.

(i) Find the polar coordinates of A. *(2 marks)*

(ii) Find, in surd form, the length of AQ. *(3 marks)*

(iii) Hence, or otherwise, explain why the line AQ is a tangent to the circle $r = 2$. *(2 marks)*

(c) Find the area of the shaded region which lies inside the circle $r = 2$ but outside the curve $r = 3 + 2\sin\theta$. Give your answer in the form $\frac{1}{6}\left(m\sqrt{3} + n\pi\right)$, where m and n are integers. *(9 marks)*