A-LEVEL
Mathematics

MFP2 Further Pure 2
Mark scheme

6360

June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or dM marks and is for accuracy
B	mark is independent of M or dM marks and is for method and accuracy
E	mark is for explanation
FT or **ft** or **F**	follow through from previous incorrect result
cao	correct answer only
cso	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, the principal examiner may suggest that we award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q 1	Solution	Mark	Total	Comment
(a)	$\dfrac{A}{2r+1}+\dfrac{B}{2r+3}$	**M1**		**and** attempt to find A or B
	$A=\dfrac{1}{4} \quad B=\dfrac{1}{4}$	**A1**	2	$\dfrac{\frac{1}{4}}{2r+1}+\dfrac{\frac{1}{4}}{2r+3}$ **OE**
(b)	$\dfrac{A}{3}+\dfrac{B}{5}-\dfrac{A}{5}-\dfrac{B}{7}+\ldots$	**M1**		clear attempt to use **method of differences** with "their" A and B
	$[k]\left\{\dfrac{1}{3}+(-1)^{n+1}\dfrac{1}{2n+3}\right\}$ **OE**	**dM1**		condone $+$, $-$, $\pm$ or $(-1)^n$ instead of $(-1)^{n+1}$; may have r for n
	$\dfrac{1}{12}+(-1)^{n+1}\dfrac{1}{4(2n+3)}$ **OE**	**A1**	3	must have n
	Total		**5**	
(b)	For **dM1 correct two** remaining terms may be on separate lines with other terms crossed out **Example 1** $\dfrac{1}{3}-\dfrac{1}{(2n+3)}$ earns **M1 dM1** **Example 2** $\dfrac{1}{12}\pm\dfrac{1}{(2n+3)}$ earns **M1 dM0** Alternative for final **A1 :** n even $\dfrac{1}{12}-\dfrac{1}{4(2n+3)}$ **;** n odd $\dfrac{1}{12}+\dfrac{1}{4(2n+3)}$			

Q 2	Solution	Mark	Total	Comment
(a)	$\alpha+\beta+\gamma=-6+3\text{i}$ **OE**	M1		**PI** by correct α
	$\alpha=-3$	A1	2	
(b) (i)	$\sum\frac{1}{\alpha\beta}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ or $\text{i}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$	M1		or $\alpha+\beta+\gamma=\text{i}\,\alpha\beta\gamma$
	$q=\frac{6-3\text{i}}{\text{i}}$ **or** $\alpha\beta\gamma=3+6\text{i}$ **OE**	A1		q , $-q$ or $\alpha\beta\gamma$ correct unsimplified **PI** by correct q
	$q=-3-6\text{i}$ **OE**	A1	3	
(ii)	$-27+9(6-3\text{i})-3p-3-6\text{i}=0$	M1		correctly substituting "their" values for α and q into equation
	$p=8-11\text{i}$ **OE**	A1	2	
(c)	$\sum\alpha^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$	M1		correct identity
	$=36-36\text{i}-9-16+22\text{i}$			
	$=11-14\text{i}$ **OE**	A1cso	2	
	Total		**9**	

(b)(i) Withhold final **A1** if $\alpha+\beta+\gamma=6-3\text{i}$ and $\alpha\beta\gamma = q$ leads to correct answer and write **FIW**

Do not treat "1" for "i" as a misread, simply an error

(b)(ii) **Alternative** $\beta\gamma=\frac{\text{"their"}\ \alpha\beta\gamma}{\alpha}=\text{"their"}-1-2\text{i}$ **;** $p=\alpha\beta+\beta\gamma+\gamma\alpha=\text{"their }\alpha\text{"}(\beta+\gamma)+\beta\gamma$ **M1 ;**
$p=8-11\text{i}$ **A1**

(c) Withhold **A1cso** if $\alpha+\beta+\gamma=6-3\text{i}$ is seen even if correct answer is given and write **FIW**

Q 3	Solution	Mark	Total	Comment
(a)	$\cosh x = \frac{e^x + e^{-x}}{2}$ **and** $\sinh x = \frac{e^x - e^{-x}}{2}$	B1		
	$1 + \frac{1}{2}(e^x + e^{-x}) = e^x - e^{-x}$			$e^x - 3e^{-x} - 2 = 0$
	$ae^{2x} + be^x + c(=0)$	M1		obtaining 3 term quadratic in e^x
	$e^{2x} - 2e^x - 3(=0)$ **OE**	A1		correct
	$(e^x - 3)(e^x + 1) \quad (=0)$	dM1		attempt at factors or correct use of formula or **PI** by both correct values 3 and –1
	$e^x = 3 \Rightarrow x = \ln 3$	A1	5	and no other value given
(b)	$\pi \int_0^{\ln 2} (1 + \cosh x)^2 \mathrm{d}x$	B1		correct expression for volume all on one line including limits, π and dx
	$\cosh^2 x = \frac{1}{2} + \frac{1}{2}\cosh 2x$ **OE**	B1		or $\cosh^2 x = \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right)$
		M1		"their" $\cosh^2 x$ term correctly integrated
	$\int = x + 2\sinh x + \frac{1}{2}x + \frac{1}{4}\sinh 2x$	A1		integral all correct
	$\pi\left(\ln 2 + 2\sinh(\ln 2) + \frac{1}{2}\ln 2 + \frac{1}{4}\sinh(2\ln 2)\right)$			
	(Volume =) $\frac{\pi}{32}(63 + 48\ln 2)$	A1	5	Allow $\frac{\pi}{32}(48\ln 2 + 63)$
	Total		**10**	

(a) If using formula they must have a simplified "correct" discriminant for "their" quadratic so if correct they must have $e^x = \frac{2 \pm \sqrt{16}}{2}$ for **dM1 ;** if factorising, "their" factors must multiply out to give "their" e^{2x} term and "their" constant term

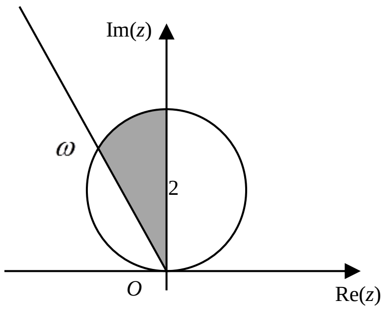
Alternatives 1 & 2 *below involve squaring which if done correctly introduce spurious solutions that must be discarded by testing whether they satisfy the original equation and so c's are likely to lose final* **A1**

Alternative 1: $1 + \cosh x = 2\sinh x \Rightarrow 1 + 2\cosh x + \cosh^2 x = 4\sinh^2 x = 4(\cosh^2 x - 1)$ **B1**
leading to quadratic in $\cosh x$ **M1 ;** $3\cosh^2 x - 2\cosh x - 5(=0)$ **A1**
$(3\cosh x - 5)(\cosh x + 1) \quad (=0)$ **dM1** "their" factors must multiply out to give "their" $\cosh^2 x$ term and "their" constant term ; **PI** by correct values $\frac{5}{3}$ and –1; **A1** for $\cosh x = \frac{5}{3} \Rightarrow x = \pm\cosh^{-1}\frac{5}{3}$ (or $\pm \ln 3$)
explaining why $x = -\ln 3$ must be rejected and giving the single solution $x = \cosh^{-1}\frac{5}{3}$ (or $\ln 3$) **.**

Alternative 2: $2\sinh x - 1 = \cosh x \Rightarrow 4\sinh^2 x - 4\sinh x + 1 = \cosh^2 x = 1 + \sinh^2 x$ **B1** leading to quadratic in $\sinh x$ **M1 ;** $3\sinh^2 x - 4\sinh x(=0)$ **A1;** $\sinh x = 0, \sinh x = \frac{4}{3}$ **dM1 (both) ;** final **A1** includes **explaining** why $x = 0$ must be rejected and giving the single solution $x = \sinh^{-1}\frac{4}{3}$ (or $\ln 3$) **.**

(b) Allow missing brackets if expanded correctly, namely $\pi\int_0^{\ln 2} 1 + 2\cosh x + \cosh^2 x \,\mathrm{d}x$ for first **B1**
Second **B1** and **M1** are available if they have different integrand (sometimes from using CSA formula)

Q 4	Solution	Mark	Total	Comment
(a)	$9k^2+17k+6$	**B1**	**1**	
(b)	When $n=1$ $LHS=2$; $RHS=2$ Therefore (formula is) true when $n=1$	**B1**		must have this explicit statement
	Assume result is true for n=k (*) Add (k + 1)th term to both sides $\sum_{r=1}^{k+1} r(2r-1)(3r-1) = \frac{1}{6}k(k+1)(9k^2-k-2)$	**M1**		adding correct (k+1)th term to RHS
	$+(k+1)(2k+1)(3k+2)$	**A1**		**both** sides correct **A0** if only RHS considered
	$=\frac{1}{6}(k+1)\left\{(9k^3-k^2-2k)+6(2k+1)(3k+2)\right\}$			correct quartic is $\frac{1}{6}\left\{9k^4+44k^3+75k^2+52k+12\right\}$
	$\frac{1}{6}(k+1)\left\{9k^3+35k^2+40k+12\right\}$	**A1**		cubic need not have all like terms collected
	$\frac{1}{6}(k+1)(k+2)(9k^2+17k+6)$			must see this line to earn final **A1**
	$\frac{1}{6}(k+1)(k+2)\left\{9(k+1)^2-(k+1)-2\right\}$	**A1**		from part **(a)**
	Hence formula is true for n=k+1 (**) and since true for n=1 , formula is true for n = 1,2,3, … [by induction] (***)	**E1**	**6**	must have (*), (**) and (***) and have earned previous 5 marks
	Total		**7**	
(b)	For **B1,** accept "n=1 RHS=LHS=2 " but must mention here or later that the result is "true when n=1" Do not allow them to simply say "true for all integers $n \ldots 1$" at the end to earn this **B1** mark. This is **B0**. Alternative to (***) is "therefore true for all positive integers n" or " so true for all integers $n \ldots 1$" etc However, "true for all $n \ldots 1$" is incorrect and immediately gets **E0** Condone LHS=$1\times1\times2+2\times3\times5+...+(k+1)(2k+1)(3k+2)$ **OE** for first **A1 but** must have **"…"** May define P(k) as the "proposition that the formula is true when n = k " and earn full marks. However, if P(k) is not defined then allow **B1** for showing P(1) is true but withhold **E1** mark.			

Q 5	Solution	Mark	Total	Comment
(a)(i)	$\tan^{-1}\dfrac{3}{\sqrt{3}}=\dfrac{\pi}{3}$	M1		or finding angle to Im(z) axis = $\dfrac{\pi}{6}$ **PI** by correct answer
	$(\arg\omega=)\dfrac{2\pi}{3}$	A1	2	
(ii)	$\left(\lvert\omega-2\mathrm{i}\rvert^2\right)=\left(-\sqrt{3}\right)^2+1^2$	M1		**PI** by correct answer
	$\lvert\omega-2\mathrm{i}\rvert=2$	A1	2	
(b)(i)		M1 A1 M1 A1		arc of circle in second quadrant circle centre at 2i (2 marked on Im(z)-axis) and touching real axis at O line from O to at least edge of circle inclined at roughly $\dfrac{\pi}{3}$ to negative real axis as drawn
		A1	5	must have earned previous 4 marks correct shading of region bounded by line, imaginary axis and circular arc
(ii)	ω marked correctly	B1	1	clear intention to be at intersection point **B0** if only line or circle drawn
(iii)	Max value = $\left\lvert\dfrac{\omega}{2}-4\mathrm{i}\right\rvert$ *or* $\left\lvert-\dfrac{\sqrt{3}}{2}+\dfrac{3\mathrm{i}}{2}-4\mathrm{i}\right\rvert$ or $\sqrt{\dfrac{\lvert\omega\rvert^2}{4}+2^2}$ etc **ACF**	M1		correct expression- distance from $\frac{1}{2}\omega$ to 4i that could be evaluated to give correct ans $d^2=\dfrac{\lvert\omega\rvert^2}{4}+4^2-4\lvert\omega\rvert\cos\dfrac{\pi}{6}$
	$=\sqrt{7}$	A1	2	
	Total		**12**	

(a)(i) **NMS** $(\theta=)\dfrac{2\pi}{3}$ **M1 A1;** $\tan^{-1}\left(-\dfrac{3}{\sqrt{3}}\right)=-\dfrac{\pi}{3}$ or sight of $-\dfrac{\pi}{3}$ earns **M1**

(b)(i) Allow freehand circle and clear "intention" to touch real axis at origin
First **A1 :** condone 2i or (0,2) or 2 clear dashes to indicate centre on Im(z) axis or radius indicated as 2 **and** circle touching real axis at O

Second **A1 :** award if angle made with negative Re(z) axis is greater than $\dfrac{\pi}{4}$

Condone circle/line as dotted lines

(iii) **NMS** max = $\sqrt{7}$ scores full marks

Q 6	Solution	Mark	Total	Comment
	$\frac{x^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-\int k\,x^2\,\frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2}\,(\mathrm{d}x)$	**M1**		Integration by parts – at least this far – (denominator may be $3+x^2$)
	$\frac{x^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-\int \frac{x^2}{2}\times\frac{1}{\sqrt{3}}\times\frac{1}{1+\frac{x^2}{3}}\,(\mathrm{d}x)$	**A1**		*or* $\frac{x^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-\int \frac{x^2}{2}\times\frac{\sqrt{3}}{3+x^2}\,(\mathrm{d}x)$ **OE**
	$\frac{x^2}{x^2+A}=1-\frac{A}{x^2+A}$ **OE**	**B1F**		or $\frac{x^2}{1+\frac{x^2}{3}}=3-\frac{3}{1+\frac{x^2}{3}}$ etc
	$\frac{x^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-\frac{\sqrt{3}}{2}x+\frac{3}{2\sqrt{3}}\times\sqrt{3}\times\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**A1**		correct unsimplified
	$I=\frac{\left(\sqrt{3}\right)^2}{2}\tan^{-1}(1)-\frac{\sqrt{3}}{2}\sqrt{3}+\frac{3}{2\sqrt{3}}\times\sqrt{3}\times\tan^{-1}(1)$	**A1**		correct unsimplified sub of limits
	$=\frac{3\pi}{8}-\frac{3}{2}+\frac{3\pi}{8}$ $=\frac{3\pi}{4}-\frac{3}{2}$	**A1**	**6**	
	Total		**6**	

Do NOT allow misread of $\frac{x}{3}$ for $\frac{x}{\sqrt{3}}$; it eases the question considerably

Alternative 1: $x=\sqrt{3}\,u$ **;** $I=\int 3u\tan^{-1}u\,\mathrm{d}u$ **=** $\frac{3}{2}u^2\tan^{-1}u-k\int\frac{u^2}{1+u^2}\,\mathrm{d}u$ **M1;** $k=\frac{3}{2}$ **A1;** $\frac{u^2}{1+u^2}=1-\frac{1}{1+u^2}$ **B1;** $\frac{3u^2}{2}\tan^{-1}u-\frac{3}{2}u+\frac{3}{2}\tan^{-1}u$ **A1 ;** then **A1 A1** as above.

Alternative 2: $x=\sqrt{3}\tan u$ **;** $\frac{\mathrm{d}x}{\mathrm{d}u}=\sqrt{3}\sec^2 u$ **;** $I=\int 3u\tan u\sec^2 u\,\mathrm{d}u$

$I=\frac{3}{2}\left[u\tan^2 u\right]-\int k\tan^2 u\,\mathrm{d}u$ **M1** $k=\frac{3}{2}$ **A1** (correct)

replacing $\tan^2 u$ by $\sec^2 u-1$ in integral **dM1 ;** $I=\frac{3}{2}\left[u\tan^2 u\right]+\frac{3}{2}u-\frac{3}{2}\tan u$ **A1 ;** then **A1 A1** as above.

Q 7	Solution	Mark	Total	Comment
(a)	$\dfrac{(1+\cosh\theta)\cosh\theta-\sinh\theta\sinh\theta}{(1+\cosh\theta)^2}$	M1		quotient rule correct
	Numerator = $\cosh\theta+1$	A1		correctly simplified
	$\times\dfrac{1+\cosh\theta}{\sinh\theta}$	dM1		
	$\text{f}\,'(\theta)=\dfrac{1}{\sinh\theta}$	A1	4	**AG** – no errors seen and $\text{f}\,'(\theta)=$ **…**
(b)(i)	$\left[1+\left(\dfrac{\text{d}y}{\text{d}x}\right)^2=\right]\ 1+\left(\dfrac{1}{x}\right)^2$	M1		condone $\int\sqrt{1+\left(\frac{1}{x}\right)^2}(\text{d}x)$ for **M1**
	$\dfrac{x^2+1}{x^2}$ or $\sqrt{\dfrac{x^2+1}{x^2}}$ or $\dfrac{\sqrt{x^2+1}}{\sqrt{x^2}}$	A1		Allow this mark but withhold final **A1** mark if $\frac{\text{d}y}{\text{d}x}$ or $\left(\frac{\text{d}y}{\text{d}x}\right)^2$ not seen
	$s=\displaystyle\int_1^{2\sqrt{2}}\frac{\sqrt{x^2+1}}{x}\,\text{d}x$	A1	3	**AG** (be convinced) - must have "$s=$", limits and dx and must have $\sqrt{x^2+1}$ in numerator
(ii)	$x=\sinh\theta\quad \text{d}x=\cosh\theta\,\text{d}\theta$ **OE**	M1		or $x=\tan\theta\quad \text{d}x=\sec^2\theta\,\text{d}\theta$
	$\displaystyle\int\frac{\cosh\theta\cosh\theta}{\sinh\theta}\,\text{d}\theta$ **(**must have $\text{d}\theta$**)**	A1		$\displaystyle\int\frac{\sec\theta\sec^2\theta}{\tan\theta}\,\text{d}\theta$
	Attempt to split into two terms using $\cosh^2\theta=\pm1\pm\sinh^2\theta$	M1		**PI** by correct split below
	$\displaystyle\int\left(\frac{1}{\sinh\theta}+\sinh\theta\right)[\text{d}\theta]$	A1		correct split and must have integral sign
	$\ln\left[\dfrac{\sinh\theta}{1+\cosh\theta}\right]+\cosh\theta$	dM1		integrating $\pm\dfrac{1}{\sinh\theta}\pm\sinh\theta$ correctly
	$s=\ \ln\left[\dfrac{2\sqrt{2}}{1+3}\right]+3-\ln\left[\dfrac{1}{1+\sqrt{2}}\right]-\sqrt{2}$	A1		correct unsimplified
	$3-\sqrt{2}+\ln\left(1+\dfrac{\sqrt{2}}{2}\right)$	A1	7	**AG** partly so be convinced
	Total		13	

(a) **Alternative:** $[\text{f}(\theta)=]\ln(\sinh\theta)-\ln(1+\cosh\theta)$ **and** one term differentiated correctly **M1**

$[\text{f}\,'(\theta)=]\ \dfrac{\cosh\theta}{\sinh\theta}-\dfrac{\sinh\theta}{1+\cosh\theta}$ **A1** $=\dfrac{(1+\cosh\theta)\cosh\theta-\sinh^2\theta}{\sinh\theta(1+\cosh\theta)}$ **dM1** (common denominator)

$\text{f}\,'(\theta)=\dfrac{1}{\sinh\theta}$ **A1** (**AG** no errors seen and $\text{f}\,'(\theta)=$ **…**)

(b)(ii) In alternative on RHS; **B1** for using $\sec^2\theta=1+\tan^2\theta$ used in numerator;

dM1 for splitting integrand $\pm\dfrac{1}{\sin\theta}\pm\sec\theta\tan\theta$ and **dM1** for integrating correctly

NB $\int(\csc\theta+\sec\theta\tan\theta)\text{d}\theta=-\ln(\csc\theta+\cot\theta)+\sec\theta$

Q 8	Solution	Mark	Total	Comment
(a)	$\cos 7\theta + \mathrm{i}\sin 7\theta = (\cos\theta + \mathrm{i}\sin\theta)^7$	B1		or $\sin 7\theta =$ Im part of $(\cos\theta + \mathrm{i}\sin\theta)^7$ **PI** by later work
	$[c^7] + 7c^6(\mathrm{i}s) + [21c^5(\mathrm{i}s)^2] + 35c^4(\mathrm{i}s)^3$ $[+35c^3(\mathrm{i}s)^4] + 21c^2(\mathrm{i}s)^5 + [7c(\mathrm{i}s)^6] + (\mathrm{i}s)^7$	M1		condone up to 2 errors in imaginary part of expansion for **M1** – ignore real terms
		A1		correct imaginary terms
	$7(1-s^2)^3(\mathrm{i}s) + 35(1-s^2)^2(\mathrm{i}s)^3$ $+21(1-s^2)(\mathrm{i}s)^5 + (\mathrm{i}s)^7$	dM1		correct use of $c^2 = 1 - s^2$ in at least two imaginary terms – i.e. $c^6 = (1-s^2)^3$ etc
	$\sin 7\theta = 7s(1-3s^2+3s^4-s^6)$ $-35s^3(1-2s^2+s^4) + 21s^5(1-s^2) - s^7$	A1		RHS correct unsimplified expansion **and** equated to $\sin 7\theta$
	$\dfrac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$	A1	6	**AG** be convinced – terms must be in this order
(b)(i)	$\sin 7\theta = 0 \Rightarrow 7\theta = (n)\pi \Rightarrow \theta = (n)\dfrac{\pi}{7}$	M1		condone no mention of "but $\sin\theta \neq 0$"
	$x = \sin^2\theta$ seen or used **so** $\sin^2\dfrac{\pi}{7}$ is a root of cubic equation	E1		must earn **M1** and have/use $x = \sin^2\theta$ **and** statement
	other roots are $\sin^2\dfrac{2\pi}{7}$ & $\sin^2\dfrac{3\pi}{7}$ **OE**	B1	3	accept $\sin^2\dfrac{4\pi}{7}$ & $\sin^2\dfrac{5\pi}{7}$ etc but **not** $\sin^2\dfrac{3\pi}{7}$ & $\sin^2\dfrac{4\pi}{7}$ etc
(ii)	Considering $\sum\dfrac{1}{\alpha}$	M1		must relate $\sin^2\dfrac{\pi}{7}$ etc to α, β, γ
	$\dfrac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$	A1		
	$= \dfrac{56/64}{7/64} = 8$	A1	3	do **not** accept $\dfrac{56}{7}$ if using this approach
	Total		**12**	

(b)(i) Condone reverse argument namely $\theta = \dfrac{\pi}{7} \Rightarrow \sin 7\theta = 0$ for **M1**

$\dfrac{\pi}{7}$ is a root of $\dfrac{\sin 7\theta}{\sin\theta} = 0$ earns **M1**

(ii) **Alternative** : put $z = 1/y$ **M1**

new equation $7z^3 - 56z^2 + 112z - 64 = 0$ **A1;** sum of these roots $= \dfrac{56}{7} = 8$ **A1**

NMS 8 scores no marks