AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

AS MATHEMATICS

Unit Further Pure 1

Wednesday 14 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve passes through the point $(4, 8)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2x + \sqrt{x}}$$

Use a step-by-step method with a step length of 0.3 to estimate the value of y at $x = 4.6$. Give your answer to four decimal places.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

Turn over ►

2 The equation $5x^2 + px + q = 0$, where p and q are constants, has roots α and $\alpha + 4$.

(a) Show that $p^2 = 20q + 400$.

[4 marks]

(b) A quadratic equation has roots α^2 and $(\alpha + 4)^2$.

(i) Find this quadratic equation, giving your answer in terms of q.

[3 marks]

(ii) Hence, or otherwise, given that the roots of this quadratic equation are equal, find the value of q.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 2**

Turn over ►

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 2

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 2**

Turn over ►

Do not write outside the box

3 It is given that $z = \mathrm{i}(1-\mathrm{i})(2+\mathrm{i})$.

(a) Show that z can be expressed in the form $k+3\mathrm{i}$, where k is an integer.

[3 marks]

(b) Hence find the values of the integers m and n such that

$$(z-\mathrm{i})^{*} - mz = n(1+4\mathrm{i})$$

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 3

Turn over ►

Do not write outside the box

4 (a) Find, in terms of c and d, the value of $\int_c^d \frac{1}{2x\sqrt{x}}\,dx$, where $0 < c < d$.

[3 marks]

(b) Hence show that only one of the following improper integrals has a finite value, and find that value:

(i) $\int_0^9 \frac{1}{2x\sqrt{x}}\,dx$;

(ii) $\int_9^\infty \frac{1}{2x\sqrt{x}}\,dx$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

Turn over ►

Do not write outside the box

5 (a) Find the general solution of the equation

$$\tan\left(2x+\frac{\pi}{2}\right)=\sqrt{3}$$

giving your answer for x in terms of π in a simplified form.

[4 marks]

(b) Use your general solution to find the possible exact values of $\sin 3x - \sin 4x$ given that $\tan\left(2x+\frac{\pi}{2}\right)=\sqrt{3}$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

6 An ellipse E_1 has equation $\frac{x^2}{16}+\frac{y^2}{4}=1$.

(a) Find the area of the rectangle whose vertices are the points of intersection of the horizontal and vertical tangents to the ellipse E_1 .

[2 marks]

(b) The ellipse E_1 can be mapped onto a circle of radius 4 by means of a one-way stretch. Write down the matrix which represents this stretch.

[2 marks]

(c) The ellipse E_1 is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ to give the ellipse E_2 .

The vertical tangents to E_2 have equations $x = 7$ and $x = -1$.

The equation of E_2 is $x^2 + 4y^2 + px + qy = 3$, where p and q are integers.

(i) Find the value of a .

[2 marks]

(ii) Find the value of p and the possible values of q .

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

Turn over ►

7 Use the relevant formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that:

(a) $\sum_{r=1}^{n}\left(r^3 - 3r\right) = \frac{n}{4}(n+a)(n+b)(n+c)$, where a, b and c are integers;

[4 marks]

(b) the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots - (2n)^2 = -n(pn+q)$$

where p and q are integers.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 7**

Turn over ►

Do not write outside the box

8 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

(a) Given that $\mathbf{C} = \begin{bmatrix} 2 & 4 \\ 6 & -2 \end{bmatrix}$ and $\mathbf{C} - 2\mathbf{D} = \mathbf{A}$, find the matrix $\mathbf{D}$.

[2 marks]

(b) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}$.

[1 mark]

(c) (i) The matrix $\mathbf{B}$ represents an anticlockwise rotation through an **obtuse** angle θ about the origin, where $\sin\theta = \frac{3}{5}$. Find the matrix $\mathbf{B}$.

[2 marks]

(ii) The point $(10,\ 15)$ is mapped onto point P under the transformation represented by $\mathbf{A}$ followed by the transformation represented by $\mathbf{B}$. Find the coordinates of P.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

Turn over ►

Do not write outside the box

9 A curve C has equation

$$y = \frac{2x^2 + 2x + 1}{(x+1)(x-3)}$$

The curve has two stationary points P and Q.

(a) Write down the equations of all the asymptotes of C.

[2 marks]

(b) The line $y = k$ intersects the curve C. Show that $4k^2 - 3k - 1 \geqslant 0$.

[5 marks]

(c) **Hence** find the length of the line segment PQ.

(No credit will be given for solutions based on differentiation.)

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 9**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 9

Turn over ►

QUESTION PART REFERENCE	Answer space for question 9

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED