Pearson
Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Friday 22 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
6690/01

You must have:
D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D2 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 5 1 5 7 2 A

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Write your answers in the D2 answer book for this paper.

1. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of four demand points, 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution to this transportation problem is required.

	1	2	3	4	Supply
A	24	32	21	34	27
B	28	31	29	37	41
C	25	41	33	35	31
D	23	32	31	36	14
Demand	33	35	25	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	1	2	3	4
A	27			
B	6	35		
C		0	25	6
D				14

Table 2

(a) Explain why a zero has been placed in cell C2 in Table 2. State the other cell in Table 2 in which the zero could have been placed.

(2)

(b) State the shadow costs clearly and enter the improvement indices into Table 3 in your answer book.

(3)

Taking the most negative improvement index to indicate the entering cell,

(c) list the stepping-stone route that should be used to obtain the next solution. You should make clear the cells that are included in your route and state your entering and exiting cells. [You do **not** need to state the next solution. You do **not** need to solve this problem.]

(2)

(Total 7 marks)

2. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	−3	2	5	−1
A plays 2	−5	3	1	−1
A plays 3	−2	5	4	2
A plays 4	2	−3	−1	4

(a) Identify the play safe strategies for each player.

(3)

(b) State, giving a reason, whether there is a stable solution to this game.

(1)

(c) Explain why the game above can be reduced to the following 3×3 game.

−3	2	5
−2	5	4
2	−3	−1

(2)

(d) Formulate the 3×3 game as a linear programming problem for player A, defining your variables clearly and writing the constraints as inequalities.

(7)

(Total 13 marks)

3. Five workers, A, B, C, D and E, are to be assigned to five tasks, 1, 2, 3, 4 and 5. Each worker must be assigned to only one task and each task must be done by only one worker.

The cost, in pounds, of assigning each worker to each task is shown in the table below. The cost of assigning worker D to task 4 is £x, where $x > 38$

	1	2	3	4	5
A	25	31	27	29	35
B	29	33	40	35	37
C	28	29	35	36	37
D	34	35	36	x	41
E	36	35	32	31	33

The total cost is to be minimised.

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the cost. You must make your method clear and show the table after each stage.

(8)

(b) Find the minimum total cost.

(1)

Workers A and D decide that they do not like the task they have been allocated and are allowed to swap tasks with each other. The other three allocations are unchanged. The cost now of allocating the five workers to the five tasks is now £5 more than the minimum cost found in (b).

(c) Calculate the value of x. You must show your working.

(2)

(Total 11 marks)

4.

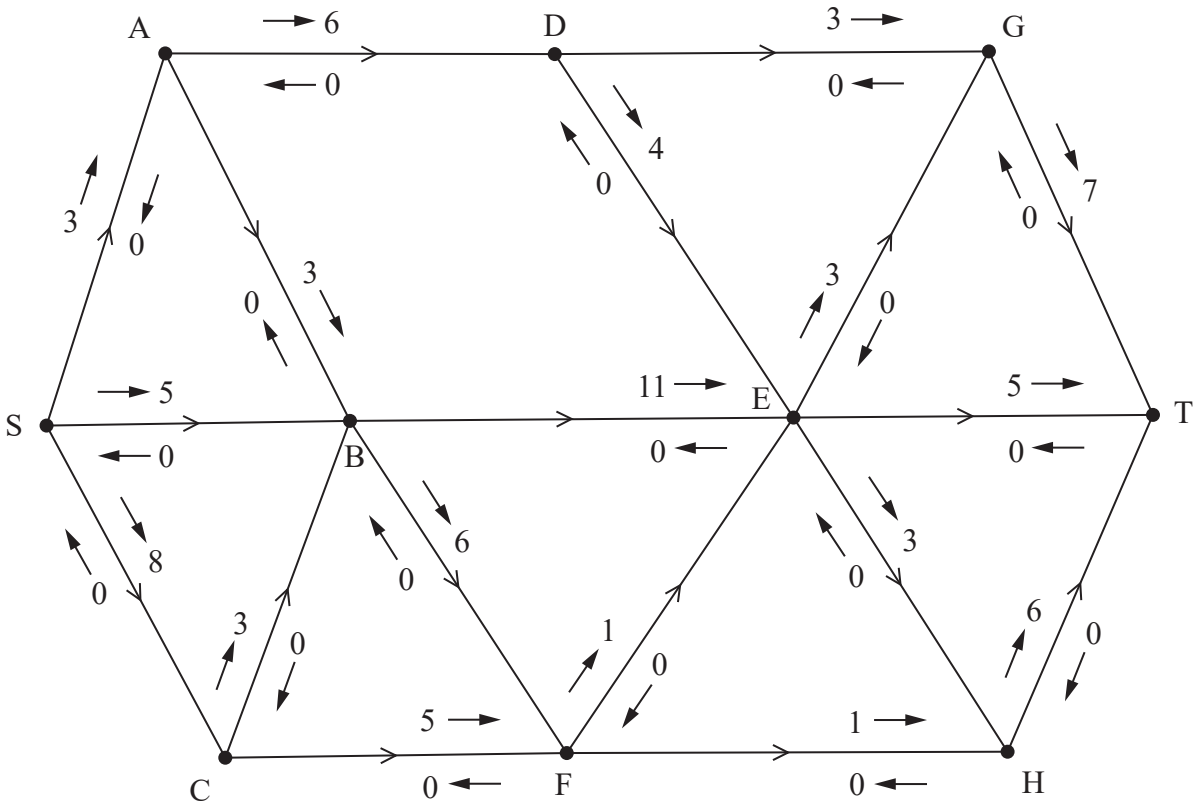

Figure 1

Figure 1 represents a system of pipes through which fluid can flow from the source node, S, to the sink node, T. The labelling procedure has been applied to Figure 1, and the numbers on the arrows, either side of each arc, show the excess capacities and potential backflows.

Currently, no fluid is flowing through the system.

(a) Calculate the capacity of the cut that passes through arcs GT, EG, DE, BE, FE and FH.

(1)

(b) Explain why arc GT can never be full to capacity when fluid is flowing through the system.

(1)

(c) Apply the labelling procedure to Diagram 1 in the answer book to show the maximum flow along SBET. State the amount that can flow along this route.

(2)

(d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(e) State the maximum flow through the system and find a cut to show that this flow is maximal.

(2)

(f) Show the maximum flow on Diagram 2 in the answer book.

(2)

(Total 12 marks)

5. The initial tableau for a linear programming problem in x, y and z is shown below. The objective function to be maximised is $P = 4x + 2y + kz$, where k is a positive constant.

Basic Variable	x	y	z	r	s	t	Value
r	−2	−6	1	1	0	0	40
s	2	3	2	0	1	0	80
t	1	2	2	0	0	1	50
P	−4	−2	$-k$	0	0	0	0

(a) Using the information in the tableau, write down the three constraints as inequalities.

(2)

(b) By increasing x, perform one complete iteration of the simplex algorithm to obtain tableau T_1 and state the row operations you use.

(4)

(c) Given that T_1 is not optimal, find an inequality for the value of k.

(1)

(d) Perform a second complete iteration of the simplex algorithm to obtain tableau T_2 and state the row operations you use.

(4)

(e) Given that T_2 is optimal, find a second inequality for the value of k.

(2)

(f) State the final value of each variable and give an expression for the final value of P in terms of k.

(2)

(g) Hence find the range of possible values of P.

(2)

(Total 17 marks)

6. Jonathan is an author who is planning his next book tour. He will visit four countries over a period of four weeks. He will visit just one country each week. He will leave from his home, S, and will only return there after visiting the four countries. He will travel directly from one country to the next. He wishes to determine a schedule of four countries to visit.

Table 1 shows the countries he could visit each week.

Week	1	2	3	4
Possible countries	A, B or C	D or E	F, G or H	I or J

Table 1

Table 2 shows the appearance fees, in £100s, he expects to earn in each country.

Country	A	B	C	D	E	F	G	H	I	J
Earnings in £100s	27	29	32	24	22	35	36	38	30	29

Table 2

Table 3 shows the cost, in £100s, of travelling between the countries.

	A	B	C	D	E	F	G	H	I	J
S	3	4	6						5	3
A				6	4					
B				5	3					
C				6	5					
D						7	6	8		
E						6	6	4		
F									5	7
G									5	7
H									6	7

Table 3

Jonathan's expected income is the value of the appearance fees minus the cost of travel. He decides to use dynamic programming to find a schedule that maximises his total expected income for these four weeks.

Complete the table in the answer book to solve this dynamic programming problem. Hence write down Jonathan's optimal expected income and state his possible optimal schedules.

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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Write your name here

Surname	Other names

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Centre Number	Candidate Number

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Total Marks

Turn over ▶

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P 5 1 5 7 2 A 0 1 1 6

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Leave blank

1.

	1	2	3	4	Supply
A	24	32	21	34	27
B	28	31	29	37	41
C	25	41	33	35	31
D	23	32	31	36	14
Demand	33	35	25	20	

Table 1

(a)

	1	2	3	4
A	27			
B	6	35		
C		0	25	6
D				14

Table 2

(b)

	1	2	3	4
A				
B				
C				
D				

Table 3

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Leave blank

Question 1 continued

(c) *You may not need to use all of these tables*

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

Q1

(Total 7 marks)

Leave blank

2.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	−3	2	5	−1
A plays 2	−5	3	1	−1
A plays 3	−2	5	4	2
A plays 4	2	−3	−1	4

Leave blank

Question 2 continued

−3	2	5
−2	5	4
2	−3	−1

Leave blank

Question 2 continued

Leave blank

Question 2 continued

(Total 13 marks)

Q2

Leave blank

3.

	1	2	3	4	5
A	25	31	27	29	35
B	29	33	40	35	37
C	28	29	35	36	37
D	34	35	36	x	41
E	36	35	32	31	33

(a) *You may not need to use all of these tables*

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

Leave blank

Question 3 continued

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

	1	2	3	4	5
A					
B					
C					
D					
E					

Q3

(Total 11 marks)

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4.

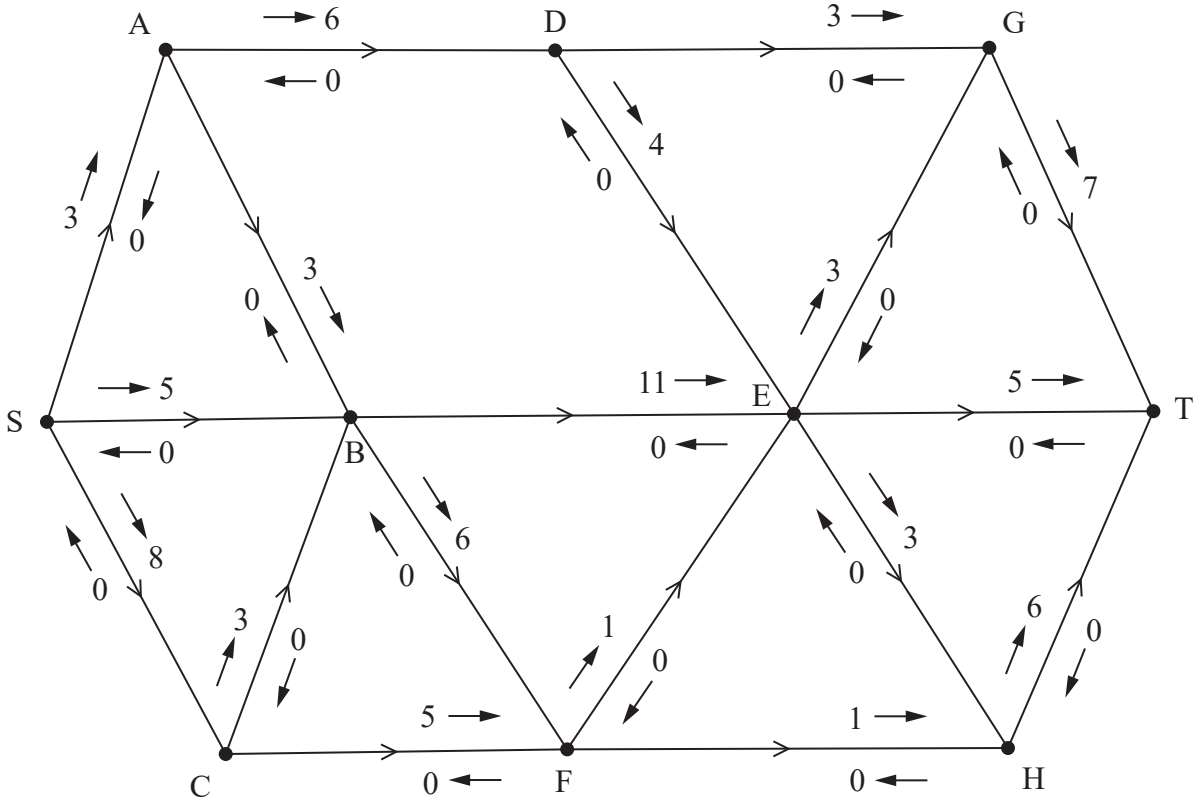

Diagram 1

Maximum flow along SBET: _____

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Question 4 continued

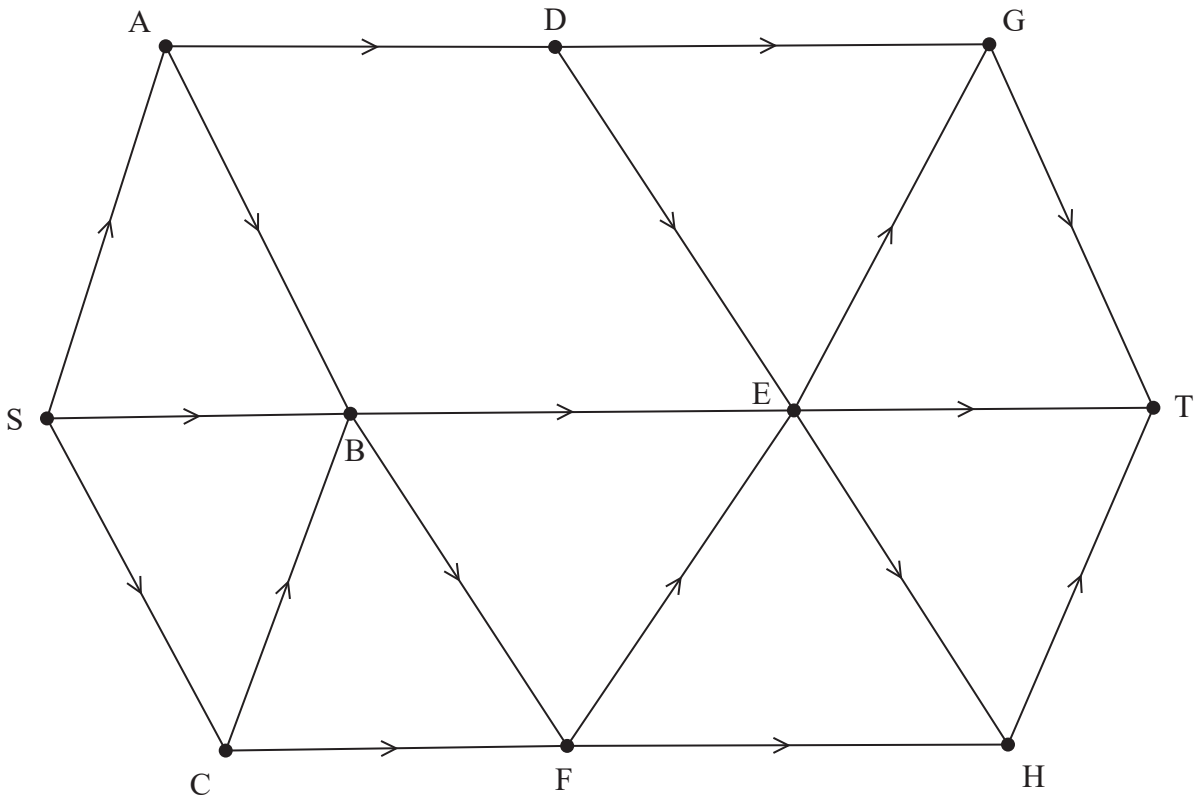

Diagram 2

(Total 12 marks)

Q4

Leave blank

5.

b.v.	x	y	z	r	s	t	Value
r	−2	−6	1	1	0	0	40
s	2	3	2	0	1	0	80
t	1	2	2	0	0	1	50
P	−4	−2	$-k$	0	0	0	0

You may not need to use all of these tableaux

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

Leave blank

Question 5 continued

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

Q5

(Total 17 marks)

Leave blank

6.

Stage	State	Action	Destination	Value
0	I	IS	S	30 − 5 = 25*

Leave blank

Question 6 continued

Stage	State	Action	Destination	Value

Leave blank

Question 6 continued

Q6

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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