

Version



**General Certificate of Education (A-level)
June 2012**

Mathematics

MD02

(Specification 6360)

Decision 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Copyright © 2012 AQA and its licensors. All rights reserved.

Copyright

AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MD02

Q	Solution	Marks	Total	Comments																		
1(a)																						
	Forward pass	M1 A1		condone one slip (follow through) all correct																		
	Backward pass	M1 A1	4	condone one slip (follow through) all correct																		
(b)	Critical paths <i>B E G K N</i> <i>D F I L N</i> Minimum completion time is 21 days	M1 A1 B1	3	first path correct second path and no others																		
(c)	Cascade diagram One of 'their' CPs correct <i>B, D, E, F, G, I, K, L, N</i> <i>A, C, H, J, M</i>	M1 A1 M1 A1 A1	 5	may be in blocks or bars (see examples) ft their CP these activities correct 3 of these with correct start and duration 3 correct with correct slack indicated all 5 correct with correct slack																		
				<table border="0"> <tr> <td></td> <td></td> <td style="text-align: right;">Slack</td> </tr> <tr> <td><i>A</i></td> <td>0 – 7</td> <td>7 – 8</td> </tr> <tr> <td><i>C</i></td> <td>0 – 5</td> <td>5 – 6</td> </tr> <tr> <td><i>H</i></td> <td>10 – 12</td> <td>12 – 14</td> </tr> <tr> <td><i>J</i></td> <td>6 – 8</td> <td>8 – 16</td> </tr> <tr> <td><i>M</i></td> <td>15 – 18</td> <td>18 – 19</td> </tr> </table>			Slack	<i>A</i>	0 – 7	7 – 8	<i>C</i>	0 – 5	5 – 6	<i>H</i>	10 – 12	12 – 14	<i>J</i>	6 – 8	8 – 16	<i>M</i>	15 – 18	18 – 19
		Slack																				
<i>A</i>	0 – 7	7 – 8																				
<i>C</i>	0 – 5	5 – 6																				
<i>H</i>	10 – 12	12 – 14																				
<i>J</i>	6 – 8	8 – 16																				
<i>M</i>	15 – 18	18 – 19																				
(d)	(Max value of x is) 10 $\Rightarrow x \leq 10$	M1 A1 cao	2	considering $J_{\text{latest}} - J_{\text{earliest}}$ (condone $x < 11$ for SC2) NMS $x \leq 10$ award M1 A1																		
	Total		14																			

MD02

Q	Solution	Marks	Total	Comments
2(a)	0 1 2 4 3 ** 3 3 ** 0 1 4 4 2 0 0 0 2 0 0 0 3 2 0 0	M1		may have large number instead of ** throughout this question
	0 1 0 4 3 ** 3 1 ** 0 1 4 2 2 0 0 0 0 0 0 0 3 0 0 0	A1		then columns
		B1	3	four lines through rows 1, 4 & 5 and column 5
				ft one slip from above for next two marks
	(b) adjustment adding 1 to double covered and - 1 to uncovered	M1		ft 'their lines and table' provided no more than one slip in earlier table must make ≤ 2 further errors for M1
	0 1 0 4 4 ** 2 0 ** 0 0 3 1 1 0 0 0 0 0 1 0 3 0 0 1	A1 \checkmark		(≤ 1 further error in adjustment)
		A1	3	correct
	(c) B4 and D5 allocated	M1		(or one complete matching ringed)
	A1 B4 C2 D5 E3	A1		one correct allocation
	A3 B4 C1 D5 E2	A1	3	2nd matching and no others 1A 2C 3E 4B 5D 1C 2E 3A 4B 5D
(d)				
	13 + 16 + 21 + 20 + 15 } or 16 + 16 + 15 + 20 + 18 } Min Total Time = 85 (min)	B1	1	
Total			10	

MD02

Q	Solution	Marks	Total	Comments																												
3(a)	<table border="1"> <tr> <td><i>P</i></td> <td><i>x</i></td> <td><i>y</i></td> <td><i>z</i></td> <td><i>s</i></td> <td><i>t</i></td> <td>value</td> </tr> <tr> <td>1</td> <td>-<i>k</i></td> <td>-6</td> <td>-5</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>1</td> <td>4</td> <td>1</td> <td>0</td> <td>11</td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>6</td> <td>0</td> <td>1</td> <td>18</td> </tr> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>	value	1	- <i>k</i>	-6	-5	0	0	0	0	2	1	4	1	0	11	0	1	3	6	0	1	18	B1 B1	2	<p>may have 1's in 's' and 't' columns interchanged</p> <p>second row correct</p> <p>third row correct</p>
	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>	value																									
1	- <i>k</i>	-6	-5	0	0	0																										
0	2	1	4	1	0	11																										
0	1	3	6	0	1	18																										
(b)	<table border="1"> <tr> <td>1</td> <td>2-<i>k</i></td> <td>0</td> <td>7</td> <td>0</td> <td>2</td> <td>36</td> </tr> <tr> <td>0</td> <td>$\frac{5}{3}$</td> <td>0</td> <td>2</td> <td>1</td> <td>$-\frac{1}{3}$</td> <td>5</td> </tr> <tr> <td>0</td> <td>$\frac{1}{3}$</td> <td>1</td> <td>2</td> <td>0</td> <td>$\frac{1}{3}$</td> <td>6</td> </tr> </table>	1	2- <i>k</i>	0	7	0	2	36	0	$\frac{5}{3}$	0	2	1	$-\frac{1}{3}$	5	0	$\frac{1}{3}$	1	2	0	$\frac{1}{3}$	6	B1 M1 A1		<p>may earn next B1 M1 if no slack variables pivot is 3 (identified or used)</p> <p>row operations (even with wrong pivot) (obtaining 0 in pivot column)</p> <p>first or second row correct</p>							
	1	2- <i>k</i>	0	7	0	2	36																									
	0	$\frac{5}{3}$	0	2	1	$-\frac{1}{3}$	5																									
0	$\frac{1}{3}$	1	2	0	$\frac{1}{3}$	6																										
		A1	4	all correct (condone multiples of rows)																												
(c)(i)	<p>($k = 1 \Rightarrow$ max reached) since there are no negative values in top row</p> <p style="text-align: right;">($P_{\max} =$) 36</p>	E1 B1✓	2	<p>provided there are no negative values in top row</p> <p>“ all positive values... ” scores E0 ft their tableau</p>																												
	<p>(ii) $k = 3$: new pivot from <i>x</i>-column is $\frac{5}{3}$ used by attempting row operation</p> <table border="1"> <tr> <td>1</td> <td>0</td> <td>0</td> <td>$\frac{41}{5}$</td> <td>$\frac{3}{5}$</td> <td>$\frac{9}{5}$</td> <td>39</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>$\frac{6}{5}$</td> <td>$\frac{3}{5}$</td> <td>$-\frac{1}{5}$</td> <td>3</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>$\frac{8}{5}$</td> <td>$-\frac{1}{5}$</td> <td>$\frac{2}{5}$</td> <td>5</td> </tr> </table> <p>Optimum reached (or $P_{\max} = \dots$)</p> <p style="text-align: right;">39</p> <p style="text-align: center;">$x = 3, y = 5, z = 0$ ($s = 0, t = 0$)</p>	1	0	0	$\frac{41}{5}$	$\frac{3}{5}$	$\frac{9}{5}$	39	0	1	0	$\frac{6}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	3	0	0	1	$\frac{8}{5}$	$-\frac{1}{5}$	$\frac{2}{5}$	5	M1 A1 ✓ A1	3	<p>ft their pivot if appropriate but must have slack variables</p> <p>first or last row correct ft one slip from their tableau in part (b) but must use correct pivot</p> <p>all correct (condone multiples of rows)</p>							
1	0	0	$\frac{41}{5}$	$\frac{3}{5}$	$\frac{9}{5}$	39																										
0	1	0	$\frac{6}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	3																										
0	0	1	$\frac{8}{5}$	$-\frac{1}{5}$	$\frac{2}{5}$	5																										
		E1 B1✓		<p>must have earned M1 and have no negative values in top row</p> <p>ft their tableau</p>																												
		B1 cso	3	must have correct final tableau																												
Total			14																													

MD02

Q	Solution	Marks	Total	Comments
4(a)(i)	Row min $-6, -3, -5, -4$ Max (row min) = -3	M1		attempt to find maximin and minimax condone one slip in values
	Col max $5, 4, -3$ Min (col max) = -3	A1		all rows min and col max values correct and max (row min) = -3 identified and min (col max) = -3 identified
	max (row min) = min (col max) = -3 \Rightarrow game has a stable solution	E1	3	full statement involving maximin and minimax and both values = -3
(ii)	Adam plays A_2 & Bill plays B_3	B1	1	
(iii)	Value of game for Bill is $+3$	B1	1	<i>Examiners must use the correct symbol for marks carried forward at the bottom of page 9 and top of page 10, ie ringed totals with arrows through them.</i>
(b)(i)	(Never play) C_2 C_2 dominated by C_1 ($-3 > -4$ and $2 > 1$)	B1	1	correct strategy stated and correct reason condone $3 < 4$ and $-2 < -1$
(ii)	$C_1: 3p - 2(1 - p)$	M1		either correct unsimplified
	$C_3: -3p + 5(1 - p)$	A1	2	both correct unsimplified { $5p - 2, 5 - 8p$ }
(iii)	$3p - 2(1 - p) = -3p + 5(1 - p)$	M1		equating their 2 expressions
	$\Rightarrow p = \frac{7}{13}$	A1	2	
(iv)	Value of game = $5 \times \frac{7}{13} - 2$			or $5 - 8 \times \frac{7}{13}$
	= $\frac{9}{13}$	B1	1	
	Total		11	

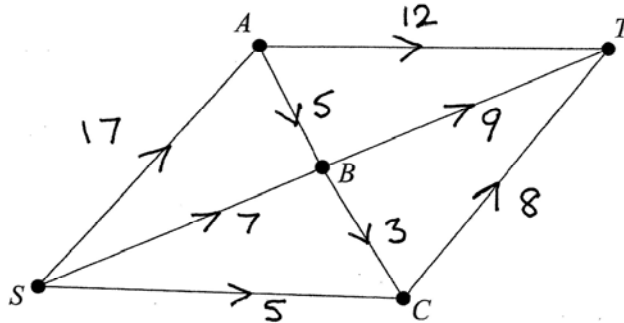
MD02

Q	Solution	Marks	Total	Comments
5(a)(i)	(<i>BAC</i> : 70, 55, 75) Least annual cost = 55	B1		£55 000
(ii)	<i>ABC</i> (involves costs 60, 75, 75) Least annual cost = 60	B1		£60 000
	<i>ABC</i> is better, since $60 > 55$	E1	3	statement & reason with both least annual costs correct
(b)	Year 3 75, 80, 60			
	Year 2			
	Calc Value			
	min (75, 75) 75 ←			
	min (70, 80) 70	M1		Finding minima for 4 of “their” pairs in Year 2
	min (55, 75) 55	A1		4 correct comparisons seen in Year 2
	min (60, 60) 60 ←			
	min (65, 80) 65 ←			
	min (80, 60) 60	A1		all values correct and comparison figures shown and correct for Years 2 and 3
	Year 1			
	A min (60, 75) 60	m1		choosing all “their” maxima from Year 2 and all “their” comparisons correct
	B min (70, 60) 60			
	C min (65, 65) 65 ←	A1cso		all correct and word “ minimum ” seen in working – (condone “min” seen once)
	Optimum order is CAB	M1		order starting with their maximum value from Year 1 in table BUT maximin must have been attempted
		A1cso	7	correct order; allow this A1cso if only error in table is omission of word “minimum” SC B1 for <i>CAB</i> if no evidence of maximin from table (or network).
	Total		10	

MD02

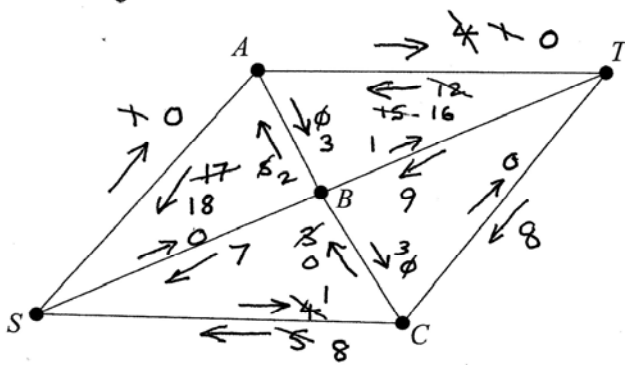
Q	Solution	Marks	Total	Comments						
6(a)(i)	18 + (0+) 10 + 3 + 5 (= 36)	B1	1							
(ii)	30, 32, 36 (missing cut values)	B3	3	B1 each value correct						
(iii)	Max flow = 29 because value of minimum cut is 29	B1 E1	2	Award B0 E1 if their min ^m cut is < 29 and min ^m value explained as max flow						
(iv)		B1 cao	1	may have <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> see alternative solution on next page </div>						
(b)(i)		M1		potential flow (forward and back) 4 pairs 'correct' including SC and AT ft their (a)(iv) provided 0 < flow < 30						
		A1 ✓		all pairs correct (condone missing 0s) ft their (a)(iv) if correct flow < 29						
		m1		one correct flow in table						
		A1		table correct <table border="1" style="margin: 10px auto;"> <tr><td>SAT</td><td>1</td></tr> <tr><td>SCT</td><td>1</td></tr> <tr><td>SCBAT</td><td>2</td></tr> </table>	SAT	1	SCT	1	SCBAT	2
	SAT	1								
SCT	1									
SCBAT	2									
	m1		If (a)(iv) flow < 29 then may score A1 for correct table giving max flow of 33 (see also the alternative solution) modifying flows (forward and back) 1 flow correct ft their initial flow							
(ii)	new max flow = 33	B1	6	modified flows all correct, including all 0s (may score A1 from a correct flow < 29 seen in (a)(iv) if final flow correct)						
		M1		6 flows correctly interpreted from their labelling procedure provided M2 or M3 scored in (b)(i) (may have AB 2, AT 16, BT 9 – see over)						
		A1	3	flow correct SC B1 if flow of 33 shown correctly but not from correct labelling procedure						
	Total		16							
	TOTAL		75							

- (iv) Indicate on the diagram below a possible flow along each edge corresponding to this maximum flow. (1 mark)



- (b) The capacities along SC and along AT are each increased by 4 litres per second.

- (i) Using your values from part (a)(iv) as the initial flow, indicate potential increases and decreases on the diagram below and use the labelling procedure to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the diagram. (6 marks)



Path	Additional Flow
SCBAT	3
SAT	1

- (ii) Use your results from part (b)(i) to illustrate the flow along each edge that gives this new maximum flow, and state the value of the new maximum flow. (3 marks)

New maximum flow is 33 litres per second

