GCE AS/A level

0976/01

MATHEMATICS – C4
Pure Mathematics

A.M. MONDAY, 16 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S14-0976-01)

1. The curve C is defined by

$$3x^3 - 5xy^2 + 2y^4 = 15.$$

The point P has coordinates (1, 2) and lies on C.
Find the equation of the **normal** to C at P. [5]

2. *(a)* Express $\dfrac{5x^2 + 7x + 17}{(x+1)^2(x-4)}$ in terms of partial fractions. [4]

(b) **Use your answer to part *(a)*** to express $\dfrac{5x^2 + 9x + 9}{(x+1)^2(x-4)}$ in terms of partial fractions. [2]

3. *(a)* Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = 3\cot x.$$ [4]

(b) (i) Express $21\sin\theta - 20\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your results to part (i) to find the greatest value of

$$\frac{1}{21\sin\theta - 20\cos\theta + 31}.$$

Write down a value for θ for which this greatest value occurs. [6]

4. The region R is bounded by the curve $y = 3 + 2\sin x$, the x-axis and the lines $x = 0$, $x = \dfrac{\pi}{4}$.

Find the volume of the solid generated when R is rotated through four right angles about the x-axis. Give your answer correct to the nearest integer. [6]

5. Expand

$$6\sqrt{1-2x} - \frac{1}{1+4x}$$

in ascending powers of x up to and including the term in x^2.
State the range of values of x for which your expansion is valid. [7]

6. The curve C has the parametric equations $x = 2t$, $y = 5t^3$. The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3.$$ [4]

(b) The tangent to C at the point P intersects C again at the point $Q(2q, 5q^3)$. Given that $p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0.$$

Hence find the value of q. [5]

7. *(a)* Find $\int x^4 \ln 2x \, \mathrm{d}x$. [4]

(b) Use the substitution $u = 10\cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)}\, \sin x \, \mathrm{d}x.$$ [4]

8. The value £V of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V.

(a) Write down a differential equation satisfied by V. [1]

(b) Show that $V = A\mathrm{e}^{kt}$, where A and k are constants. [3]

(c) The value of the investment after 2 years is £292 and its value after 28 years is £637.

(i) Show that $k = 0{\cdot}03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

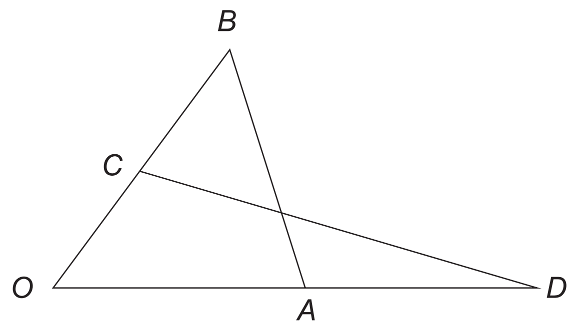
TURN OVER

9. *(a)* The vectors $\mathbf{p}$ and $\mathbf{q}$ are given by

$$\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$
and
$$\mathbf{q} = 5\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}.$$

Find the angle between $\mathbf{p}$ and $\mathbf{q}$. [4]

(b) In the diagram below, the points *O*, *A*, *B*, *C* and *D* are such that *A* is the mid-point of *OD* and *C* is the mid-point of *OB*.



Taking *O* as the origin, the position vectors of *A* and *B* are denoted by $\mathbf{a}$ and $\mathbf{b}$ respectively.

(i) Show that $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$.

Hence show that the vector equation of the line *CD* may be expressed in the form

$$\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}.$$

The vector equation of the line *L* may be expressed in the form

$$\mathbf{r} = \frac{1}{3}\mu\mathbf{a} + \frac{1}{3}(\mu - 1)\mathbf{b}.$$

The lines *CD* and *L* intersect at the point *E*.

(ii) By giving λ and μ appropriate values, or otherwise, show that *E* has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

(iii) Give a geometrical interpretation of the fact that *E* has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$. [7]

10. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

END OF PAPER