RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for January 2011

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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1	**(i)**	First two terms are $1-\frac{1}{2}x........$	B1	
		Third term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}[(-x)^2 \text{ or } x^2 \text{ or } -x^2]$	M1	
		$= -\frac{1}{8}x^2$	A1 **3**	$-\frac{1}{8}x^2$ without work $\rightarrow$ M1 A1
	(ii)	Attempt to replace x by $2y-4y^2$ or $2y+4y^2$	M1	or write as $1-\left(2y-4y^2 \text{ or } 2y+4y^2\right)$
		First two terms are $1-y$	B1	
		Third term = $+\frac{3}{2}y^2$ or $\surd\ (4b+2)y^2$	A1√ **3**	where b = cf$\left(x^2\right)$ in part (i)
			6	
2	**(i)**	$A(x-2)+B=7-2x$	M1	or $A(x-2)^2+B(x-2)=(7-2x)(x-2)$
		$A=-2$	A1	
		$B=3$	A1 **3**	
	(ii)	$\int \frac{A}{x-2}\,dx = \left(A \text{ or } \frac{1}{A}\right)\ln(x-2)$	B1	Accept $\ln\lvert x-2\rvert$, $\ln\lvert 2-x\rvert$, $\ln(2-x)$
		$\int \frac{B}{(x-2)^2}\,dx = -\left(B \text{ or } \frac{1}{B}\right).\frac{1}{x-2}$	B1	Negative sign <u>is</u> required
		Correct f.t. of A & B; $A\ln(x-2)-\frac{B}{x-2}$	B1√	Still accept lns as before
		Using limits = $-2\ln 3+2\ln 2+\frac{1}{2}$ ISW	B1 **4**	<u>No</u> indication of ln(negative)
			7	
3	**(i)**	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x=\frac{1}{\cos x}$
		Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv-v\,du}{v^2}$ & wrong trig signs
		Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
		Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. <u>Not</u> just = $\sec x.\tan x$
	(ii)	Use $\cos 2x = +/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm\left(\cos^2 x-\sin^2 x\right)$
		Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
		Evidence that $\frac{\tan x}{\cos x}=\sec x\tan x$ or $\int\frac{\tan x}{\cos x}\,dx=\sec x$	B1	irrespective of any const multiples
		$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for x except final line
			8	

4	(i)	Attempt to use $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dt}.\frac{dt}{dx}$	M1		Not just quote formula
		$\frac{4}{2t}$ or $\frac{2}{t}$	A1	2	
	(ii)	Subst $t=4$ into their (i), invert & change sign	M1		
		Subst $t=4$ into (x,y) & use num grad for tgt/normal	M1		
		$y=-2x+52$ AEF CAO (no f.t.)	A1	3	Only the eqn of normal accepted
	(iii)	Attempt to eliminate t from the 2 given equations	M1		
		$x=2+\frac{y^2}{16}$ or $y^2=16(x-2)$ AEF ISW	A1	2	Mark at earliest acceptable form.
					7
5	(i)	Attempt to connect dx and du	M1		Including $\frac{du}{dx}=$ or $du=...dx$; not dx = du
		$5-x=4-u^2$	B1		perhaps in conjunction with next line
		Show $\int\frac{4-u^2}{2+u}.2u\,du$ reduced to $\int 4u-2u^2\,du$ **AG**	A1		In a fully satisfactory & acceptable manner
		Clear explanation of why limits change	B1		e.g. when x = 2, u = 1 <u>and</u> when x = 5, u = 2
		$\frac{4}{3}$	B1	5	not dependent on any of first 4 marks
	(ii)(a)	$5-x$	*B1	1	Accept $4-x-1=5-x$ (this is not **AG**)
	(b)	Show reduction to $2-\sqrt{x-1}$	dep*B1		
		$\int\sqrt{x-1}\,dx=\frac{2}{3}(x-1)^{\frac{3}{2}}$	B1		Indep of other marks, seen anywhere in (b)
		$\left(10-\frac{2}{3}.8\right)-\left(4-\frac{2}{3}\right)=\frac{4}{3}$ or $4\frac{2}{3}-3\frac{1}{3}=\frac{4}{3}$	B1	3	Working must be shown
					9
6	(i)	Work with correct pair of direction vectors	M1		
		Demonstrate correct <u>method</u> for finding scalar product	M1		Of <u>any</u> two 3x3 vectors rel to question
		Demonstrate correct <u>method</u> for finding modulus	M1		Of <u>any</u> vector relevant to question
		24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)	A1	4	Mark earliest value, allow trunc/rounding
	(ii)	Attempt to set up 3 equations	M1		Of type $3+2s=5, 3s=3+t, -2-4s=2-2t$
		Find correct values of $(s,t)=(1,0)$ or $(1,4)$ or $(5,12)$	A1		Or 2 diff values of s (or of t)
		Substitute their (s,t) into equation not used	M1		and make a relevant deduction
		<u>Correctly</u> demonstrate failure	A1	4	dep on all 3 prev marks
	(iii)	Subst their (s,t) from first 2 eqns into new 3rd eqn	M1		New 3rd eqn of type $a-4s=2-2t$
		$a=6$	A1	2	
					10

Q		Answer	Marks		Guidance
7		Attempt parts with $u = x^2+5x+7$, $dv = \sin x$	M1		as far as $f(x) +/- \int g(x)\,dx$
		1st stage = $-(x^2+5x+7)\cos x + \int (2x+5)\cos x\,dx$	A1		signs need not be amalgamated at this stage
		$\int (2x+5)\cos x\,dx = (2x+5)\sin x - \int 2\sin x\,dx$	B1		indep of previous A1 being awarded
		$= (2x+5)\sin x + 2\cos x$	B1		
		I = $-(x^2+5x+7)\cos x + (2x+5)\sin x + 2\cos x$	A1		WWW
		(Substitute $x=\pi$) −(Substitute $x=0$)	M1		An attempt at subst $x = 0$ must be seen
		$\pi^2+5\pi+10$ WWW **AG**	A1	7	
					7
8	(i)	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(-5xy) = (-)(5)x\frac{dy}{dx} + (-)(5)y$	M1		i.e. reasonably clear use of product rule
		LHS completely correct $4x - 5x\frac{dy}{dx} - 5y + 2y\frac{dy}{dx} (=0)$	A1		Accept " $\frac{dy}{dx} =$ " provided it is not used
		Substitute $\frac{dy}{dx} = \frac{3}{8}$ or solve for $\frac{dy}{dx}$ & then equate to $\frac{3}{8}$	M1		Accuracy not required for "solve for $\frac{dy}{dx}$"
		Produce $x = 2y$ WWW **AG** (Converse acceptable)	A1	5	Expect $17x = 34y$ and/or $\frac{dy}{dx} = \frac{5y-4x}{2y-5x}$
	(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1		
		Produce either $x^2 = 36$ or $y^2 = 9$	A1		
		AEF of $(\pm 6, \pm 3)$	A1	3	ISW Any correct format acceptable
					8
9	(i)	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,dx = \int q\,dt$	M1		Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
		$\int \frac{1}{(x-8)^{1/3}}\,dx = k(x-8)^{2/3}$	A1		k const
		All correct (+ c)	A1		
		For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
		Correct corresponding value of c from correct eqn	A1		
		Subst their c & $x = 35$ back into eqn	M1		
		$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	7	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
	(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
		Substitute $x = 8$ back into equation containing numeric 'c'	M1		
		$t = 6$	A1	3	
					10

1 When an acceptable answer has been obtained, ignore subsequent working (ISW) unless stated otherwise.

2 Ignore working which has no relevance to question as set; e.g. in Qu.1, ignore all terms in x^3 etc.

3 The 'M' marks are awarded if it is clear that candidate is attempting to do what he/she should be doing.

4 If an ans is given (**AG**), working must be checked minutely as answer shown will nearly always be 'correct'. More reasoning/explanation is generally required than when the answer is not given.

Comments or Alternative methods

Question 1(ii)

Beware: there are often double mistakes leading to the correct terms – errors invalidate marks.

Question 2(ii)

For the first 2 marks, we're really testing $\int \frac{1}{x-2}\,dx$ and $\int \frac{1}{(x-2)^2}\,dx$; this is why we accept $\frac{1}{A}$ and/or $-\frac{1}{B}$.

For the 1st & 3rd marks, accept $\ln(2-x)$ as these are the indef integ stages. At final, definite, stage, it will be penalised..

'Exact value' is required; so 0.0945…. without equivalent log version $\rightarrow B0$ $2\ln 2 - 3\ln 3$ need not be simplified.

Question 4

Allow marks for part (iii) to be awarded at any stage of question.

So, if the Cartesian equation is worked out first of all, then award marks in part (i) as follow:

if cart. eqn is found in the form $x = f(y)$, award M1 for finding $\frac{dx}{dy}$, inverting & subst $y = 4t$ (in either order)

if cart. eqn is found in the form $y = g(x)$, award M1 for finding $\frac{dy}{dx}$ and substituting $x = 2 + t^2$

and, finally, A1 as in main scheme.

Question 5(i)

The problem here will centre on how the candidate manipulates the equation $u = \sqrt{x-1}$ to get x in terms of u. He/she could get $x = u^2 + 1$ (correct) or, perhaps, $x = u^2 - 1$ or $x = 1 - u^2$ (incorrect) or some other incorrect version.

The 1st, 4th & 5th marks in part (i) are unaffected by the correctness or otherwise of this manipulation. However, any error seen must destroy the 2nd and 3rd marks – but candidates can still score 3 of the 5 marks.

For the A1, there must be some evidence of reduction to the given answer; the one main case that we are not accepting is where $\frac{8u - 2u^3}{2+u}$ is said to be $4u - 2u^2$ without any supporting evidence; long division will suffice; or if $8u - 2u^3$ is said to be $(2+u)(4u - 2u^2)$, then we will accept (as multiplication can easily be checked in the head whereas division is not reckoned to be). Note that '2' into '$8u$' gives '$4u$' and 'u' into '$-2u^3$' gives '$-2u^2$'.

Question 5(ii)(a)

This is just a '1' mark part so we give 1 or 0 purely dependent on the answer and we ignore any sloppy working.

A candidate writing $4 - x - 1 = 3 - x$ will be awarded 0 marks; however, another candidate writing $4 - x - 1 = 5 - x$ will be awarded the B1 mark. This is not an **AG** so the candidate does not know the required answer.

Question 6(i)

For demonstrating correct method for finding scalar product, I expect to see at least $2/3$ of the working correct.

Likewise for modulus: examine either vector, $\sqrt{2^2+3^2-4^2}$ will score M1 {$\frac{2}{3}$ correct, prob $\sqrt{29}$ will follow anyway}

Question 6(ii)

Occasionally candidates do not follow a 'sensible' method. However, the first M1 is always standard. The remaining 3 marks must be awarded for convincing arguments and/for accurate results.

Question 7

This is a question where signs are crucial and where the given answer may be obtained even with errors in the working; also the fact that the answer is **AG** means that many candidates will state it on the final line.

Using the standard method, 3 marks out of the 7 are fixed (the 2 @ M1 and the final A1) but the other 4 marks depend on the capability of the candidate to integrate sin *x* and cos *x*.

If he/she uses cos *x* for the integral of sin *x*, candidate should get $-(\text{our version of 1st main stage})$, so that's A0 but he/she still has to integrate $(2x+5)\cos x$ for the 2nd stage. Admittedly he/she may then make a further mistake when integrating cos *x* but the 2 @ B1 are available. These 2 marks are an independent pair and only depend on the integral of $(2x+5)\cos x$ being attempted. Whether it's the integral of $(2x+5)\cos x$ or of $-(2x+5)\cos x$ is immaterial. This gives a maximum of 4 out of 7 if sin *x* is incorrectly integrated.

Even though I have bracketed the 3 terms as (x^2+5x+7), we can expect some candidates to multiply out as 3 separate integrals., $\int x^2 \sin x \, dx$ and $\int 5x \sin x \, dx$ and $\int 7 \sin x \, dx$

Their equivalent 1st stages are:

$-x^2\cos x + \int 2x\cos x \, dx$; $-5x\cos x + \int 5\cos x \, dx$; $-7\cos x$ **M1 + A1**

Their equivalent 2nd stages are:

$2x\sin x + 2\cos x$ **B1** $5\sin x$ **B1**

To obtain the corresponding marks, all components must be correct.

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