

# Mathematics

Advanced GCE A2 7890 - 2

Advanced Subsidiary GCE AS 3890 - 2

## Mark Schemes for the Units

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**January 2010**

**3890-2/7890-2/MS/10J**

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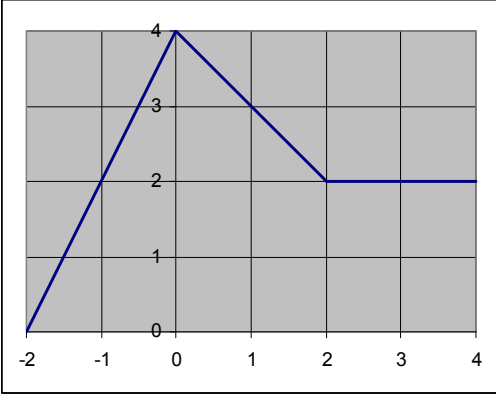
**Advanced Subsidiary GCE Mathematics (3890)**  
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# 4721 Core Mathematics 1

1	$[(x-6)^2 - 36] + 1$ $= (x-6)^2 - 35$	<b>B1</b> $(x-6)^2$ <b>M1</b> $q = 1 - (\text{their } p)^2$ <b>A1</b> 3 $q = -35$	
			3

2 (i)		<b>B1</b> For $x < 0$ , straight line joining $(-2, 0)$ and $(0, 4)$  <b>B1</b> 2 For $x > 0$ , line joining $(0, 4)$ to $(2, 2)$ and horizontal line joining $(2, 2)$ and $(4, 2)$	
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(ii)	Translation 1 unit right parallel to $x$ axis	<b>B1</b> <b>B1</b> 2 Allow: 1 unit right, 1 along the $x$ axis, 1 in <b>direction</b> , allow vector notation e.g. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , 1 unit horizontally	
			4

3	$\frac{dy}{dx} = 3x^2 - 8x$	<b>M1</b> Attempt to differentiate (one of $3x^2, -8x$ ) <b>A1</b> Correct derivative	
	When $x = 2$ , $\frac{dy}{dx} = -4$	<b>M1</b> Substitutes $x = 2$ into their $\frac{dy}{dx}$ <b>A1</b>	
	$\therefore$ Gradient of normal to curve = $\frac{1}{4}$	<b>B1 ft</b> Must be numerical $= -1 \div \text{their } m$	
	$y + 1 = \frac{1}{4}(x - 2)$	<b>M1</b> Correct equation of straight line through $(2, -1)$ , any non-zero numerical gradient	
	$x - 4y - 6 = 0$	<b>A1</b> 7 Correct equation in required form	
			7

4	(i) $m = 4$	B1	1	May be embedded
	(ii) $6p^2 = 24$ $p^2 = 4$ $p = 2$ or $p = -2$	M1 A1 A1	3	$(\pm)6p^2 = 24$ or $36p^4 = 576$
	(iii) $5^{2n+4} = 25$	M1		Addition of indices as powers of 5
	$\therefore 2n + 4 = 2$ $n = -1$	M1 A1	3	Equate powers of 5 or 25
5	$k = \sqrt{x}$ $k^2 - 8k + 13 = 0$	M1*		Use a substitution to obtain a quadratic (may be implied by squaring or rooting later) or factorise into 2 brackets each containing $\sqrt{x}$
	$k - 4 = \pm\sqrt{3}$ or $k = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 13}}{2}$	M1 dep A1		Correct method to solve resulting quadratic
	$k = 4 \pm \sqrt{3}$	A1		$k = 4 \pm \sqrt{3}$ or $k = \frac{8 \pm \sqrt{12}}{2}$ or $k = 4 \pm \frac{\sqrt{12}}{2}$
	$\therefore x = (4 + \sqrt{3})^2$ or $x = (4 - \sqrt{3})^2$	M1 M1		Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)
	$x = 19 \pm 8\sqrt{3}$ or $19 \pm 4\sqrt{12}$	A1	7	
6	(i) $\frac{dy}{dx} = 2x$ When $x = 1$ , $\frac{dy}{dx} = 2$	B1* B1 dep	2	
	(ii) $\frac{a^2 + 5 - 6}{a - 1} = 2.3$ $a^2 - 2.3a + 1.3 = 0$ $(a - 1.3)(a - 1) = 0$ $a = 1.3$	M1 A1 M1 A1	4	uses $\frac{y_2 - y_1}{x_2 - x_1}$ correct expression correct method to solve a quadratic or correct factorisation and cancelling to get $a + 1 = 2.3$ 1.3 only

<b>Alternative method:</b>			
Equation of straight line through (1,6) with $m = 2.3$ found then			
$a^2 + 5 = 2.3a + "c"$ seen <b>M1</b>			
with $c = 3.7$ <b>A1</b>			
then as main scheme			
	(iii)	A value between 2 and 2.3	<b>B1</b> 1 7 $2 < \text{value} < 2.3$ (strict inequality signs)
7	(i)	(a) Fig 3 (b) Fig 1 (c) Fig 4	<b>B1</b> <b>B1</b> <b>B1</b> 3
	(ii)	$-(x-3)^2$	<b>M1</b> Quadratic expression with correct $x^2$ term and correct $y$ -intercept and/or roots for <b>their unmatched diagram</b> (e.g. negative quadratic with $y$ -intercept of $-9$ or root of 3 for Fig 2)
		$y = -(x-3)^2$	<b>A1</b> 2 5 Completely correct equation for <b>Fig 2</b>
8	(i)	Centre $(-3, 2)$ $(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$ $r^2 = 17$ $r = \sqrt{17}$	<b>B1</b> <b>M1</b> Correct method to find $r^2$ <b>A1</b> 3 Correct radius
	(ii)	$x^2 + (3x+4)^2 + 6x - 4(3x+4) - 4 = 0$	<b>M1*</b> substitute for $x/y$ or attempt to get an equation in 1 variable only
			<b>A1</b> correct unsimplified expression
		$10x^2 + 18x - 4 = 0$ $(5x-1)(x+2) = 0$	<b>A1</b> obtain correct 3 term quadratic <b>M1</b> correct method to solve their quadratic <b>dep</b>
		$x = \frac{1}{5}$ or $x = -2$	<b>A1</b>
		$y = \frac{23}{5}$ or $y = -2$	<b>A1</b> 6 <b>SR</b> If A0 A0, one correct pair of values, spotted or from correct factorisation <b>www B1</b>
			9
9	(i)	$f'(x) = -x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$	<b>M1</b> Attempt to differentiate
			<b>A1</b> $-x^{-2}$ or $-\frac{1}{2}kx^{-\frac{1}{2}}$ <b>www</b>
			<b>A1</b> 3 Fully correct expression

(ii)	$f''(x) = 2x^{-3} + \frac{1}{4}x^{-\frac{3}{2}}$	<b>M1</b>	Attempt to differentiate their $f'(x)$
		<b>A1 ft</b>	One correctly differentiated term
		<b>A1</b>	Fully correct expression <b>www</b> in either part of the question
	$f''(4) = \frac{2}{4^3} + \frac{1}{4} \cdot \frac{1}{8}$ $= \frac{1}{16}$	<b>M1</b>	Substitution of $x = 4$ into their $f''(x)$
		<b>A1</b>	oe single fraction <b>www</b> in either part of the question
<b>10</b>	$(-30)^2 - 4 \times k \times 25k = 0$	<b>M1</b>	Attempts $b^2 - 4ac$ involving $k$
	$900 - 100k^2 = 0$	<b>M1</b>	States their discriminant = 0
	$k = 3$	<b>B1</b>	
	or $k = -3$	<b>B1</b>	
		<b>4</b> <b>4</b>	
<b>11</b>	(i) $P = 2 + x + 3x + 2 + 5x + 5x$ $= 14x + 4$	<b>M1</b>	Adds lengths of all 4 edges with attempt to use Pythagoras to find the missing length
		<b>A1</b>	<b>2</b>
			May be left unsimplified
	(ii) Area of rectangle = $3x(2 + x) = 6x + 3x^2$	<b>M1</b>	Correct method – splitting or formula for area of trapezium
	Area of triangle = $\frac{1}{2}(3x)(4x) = 6x^2$		
	Total area = $9x^2 + 6x$	<b>A1</b>	<b>2</b>
			Convincing working leading to given expression <b>AG</b>
	(iii) $14x + 4 \geq 39$	<b>B1 ft</b>	ft on their expression for $P$ from (i) unless restarted in (iii). (Allow $>$ )
	$\frac{5}{2}$	<b>B1</b>	o.e. (e.g. $\frac{35}{14}$ ) soi by subsequent working
	$9x^2 + 6x < 99$	<b>B1</b>	
	$3x^2 + 2x - 33 < 0$		
	$(3x + 11)(x - 3) < 0$	<b>M1</b>	Allow $\leq$
	$\left(-\frac{11}{3} < \right)x < 3$		Correct method to find critical values
		<b>B1</b>	$x < 3$ identified
		<b>M1</b>	root from linear $< x <$ upper root from quadratic
	$\therefore \frac{5}{2} \leq x < 3$	<b>A1</b>	<b>7</b> <b>11</b>
			Fully correct including inequality signs or exact equivalent in words <b>cwo</b>
<b>Total</b>		<b>72</b>	

## 4722 Core Mathematics 2

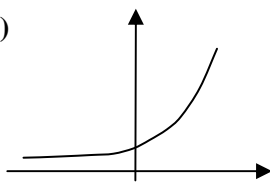
1	(i) $2(1 - \cos^2 x) = 5\cos x - 1$ $2\cos^2 x + 5\cos x - 3 = 0$ <b>A.G.</b>	M1 A1	Use $\sin^2 x = 1 - \cos^2 x$ 2 Show given equation correctly
-----			
	(ii) $(2\cos x - 1)(\cos x + 3) = 0$  $\cos x = \frac{1}{2}$ $x = 60^\circ$ $x = 300^\circ$	M1 M1 A1 A1√	Recognise equation as quadratic in $\cos x$ and attempt recognisable method to solve Attempt to find $x$ from root(s) of quadratic Obtain $60^\circ$ or $\frac{\pi}{3}$ or 1.05 rad 4 Obtain $300^\circ$ only (or $360^\circ -$ their $x$ ) and no extra in range <b>SR</b> answer only is B1 B1
<b>6</b>			
-----			
2	(i) $\int (6x - 4)dx = 3x^2 - 4x + c$  $y = 3x^2 - 4x + c \Rightarrow 5 = 12 - 8 + c$ $\Rightarrow c = 1$ Hence $y = 3x^2 - 4x + 1$	M1*  A1 M1dep* A1	Attempt integration (inc. in power for at least one term)  Obtain $3x^2 - 4x$ (or unsimplified equiv), with or without $+ c$ Use (2, 5) to find $c$ 4 Obtain $y = 3x^2 - 4x + 1$
-----			
	(ii) $3p^2 - 4p + 1 = 5$  $3p^2 - 4p - 4 = 0$ $(p - 2)(3p + 2) = 0$ $p = -\frac{2}{3}$	M1* M1dep* A1	Equate their $y$ (from integration attempt) to 5 Attempt to solve three term quadratic 3 Obtain $p = -\frac{2}{3}$ (allow any variable) from correct working; condone $p = 2$ still present, but A0 if extra incorrect solution
<b>7</b>			
-----			
3	(i) $(2 - x)^7 = 128 - 448x + 672x^2 - 560x^3$	M1  A1 A1 A1	Attempt (at least) two relevant terms – product of binomial coeff, 2 and $x$ (or expansion attempt that considers all 7 brackets) Obtain $128 - 448x$ Obtain $672x^2$ 4 Obtain $-560x^3$
-----			
	(ii) $-560 \times (\frac{1}{4})^3 = -\frac{35}{4}$	M1 A1	Attempt to use coeff of $x^3$ from (i), with clear intention to cube $\frac{1}{4}$ 2 Obtain $-\frac{35}{4}$ ( $w^6$ ), (allow $\frac{35}{4}$ from $+560x^3$ in (i))
<b>6</b>			



4	(i)	$\int_3^5 \log_{10}(2+x) dx \approx \frac{1}{2} \times \frac{1}{2} \times (\log 5 + 2 \log 5.5 +$	M1	Attempt y-coords for at least 4 of the correct 5 x-coords only
		$2 \log 6 + 2 \log 6.5 + \log 7)$	M1	Use correct trapezium rule, any $h$ , to find area between $x = 3$ and $x = 5$
		$\approx 1.55$	M1	Correct $h$ (soi) for their $y$ -values
			A1	Obtain 1.55
-----				
	(ii)	$\int_3^5 \log_{10}(2+x)^{\frac{1}{2}} dx = \frac{1}{2} \int_3^5 \log_{10}(2+x) dx$	B1√	Divide by 2, or equiv, at any stage to obtain 0.78 or 0.77,
		$\approx \frac{1}{2} \times 1.55$		following their answer to (i)
		$\approx 0.78$	B1	Explicitly use $\log \sqrt{a} = \frac{1}{2} \log a$ on a single term
<b>6</b>				
5		$\int_1^3 \{(11-9x^{-2}) - (x^2+1)\} dx = [9x^{-1} - \frac{1}{3}x^3 + 10x]_1^3$	M1	Attempt subtraction (correct order) at any point
		$= (3-9+30) - (9-\frac{1}{3}+10)$	M1	Attempt integration – inc. in power for at least one term
		$= 24 - 18\frac{2}{3}$	A1	Obtain $\pm (-\frac{1}{3}x^3 + 10x)$ or $11x$ and $\frac{1}{3}x^3 + x$
		$= 5\frac{1}{3}$	M1	Obtain remaining term of form $kx^{-1}$
		<b>OR</b>	A1	Obtain $\pm 9x^{-1}$ or any unsimplified equiv
		$[11x + 9x^{-1}]_1^3 - [\frac{1}{3}x^3 + x]_1^3$	M1	Use limits $x = 1, 3$ – correct order & subtraction
		$= [(33+3) - (11+9)] - [(9+3) - (\frac{1}{3}+1)]$	A1	Obtain $5\frac{1}{3}$ , or exact equiv
		$= 16 - 10\frac{2}{3}$		
		$= 5\frac{1}{3}$		
<b>7</b>				
6	(i)	$f(-3) = 0 \Rightarrow -54 + 9a - 3b + 15 = 0$	M1	Attempt $f(-3)$ and equate to 0, or equiv method
		$3a - b = 13$	A1	Obtain $3a - b = 13$ , or unsimplified equiv
		$f(2) = 35 \Rightarrow 16 + 4a + 2b + 15 = 35$	M1	Attempt $f(2)$ and equate to 35, or equiv method
		$2a + b = 2$	A1	Obtain $2a + b = 2$ , or unsimplified equiv
		Hence $a = 3, b = -4$	M1	Attempt to solve simultaneous eqns
			A1	Obtain $a = 3, b = -4$
-----				
(ii)	$f(x) = (x+3)(2x^2 - 3x + 5)$		M1	Attempt complete division by $(x+3)$ , or equiv
			A1	Obtain $2x^2 - 3x + c$ or $2x^2 + bx + 5$ , from correct $f(x)$
		ie quotient is $(2x^2 - 3x + 5)$	A1	Obtain $2x^2 - 3x + 5$ (state or imply as quotient)
<b>9</b>				

7	(i) $13^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos \theta$  $\cos \theta = 0.4536$ $\theta = 1.10$ <b>A.G.</b>	M1  A1	2	Attempt to use correct cosine rule in $\Delta ABC$  Obtain 1.10 radians (allow 1.1 radians) <b>SR</b> B1 only for verification of 1.10, unless complete method
-----				
	(ii) arc $EF = 4 \times 1.10 = 4.4$  perimeter = $4.4 + 10 + 13 + 6$  $= 33.4$ cm	B1  M1  A1	3	State or imply $EF = 4.4$ cm (allow $4 \times 1.10$ ) Attempt perimeter of region - sum of arc and three sides with attempt to subtract 4 from at least one relevant side Obtain 33.4 cm
-----				
	(iii) area $AEF = \frac{1}{2} \times 4^2 \times 1.1$  $= 8.8$ area $ABC = \frac{1}{2} \times 10 \times 14 \times \sin 1.1$  $= 62.4$  hence total area = $53.6 \text{ cm}^2$	M1  A1 M1  A1  A1	5	Attempt use of $(\frac{1}{2})r^2\theta$ , with $r = 4$ and $\theta = 1.10$ Obtain 8.8 Attempt use of $(\frac{1}{2})absin\theta$ , sides consistent with angle used Obtain 62.4 or better (allow 62.38 or 62.39) Obtain total area as $53.6 \text{ cm}^2$
				<b>10</b>
<hr/>				
8	(i) $u_5 = 8 + 4 \times 3$  $= 20$ <b>A.G.</b>	M1  A1	2	Attempt $a + (n - 1)d$ or equiv inc list of terms Obtain 20
-----				
	(ii) $u_n = 3n + 5$ ie $p = 3, q = 5$	B1  B1	2	Obtain correct expression, poss unsimplified, eg $8 + 3(n - 1)$ Obtain correct $3n + 5$ , or $p = 3, q = 5$ stated
-----				
	(iii) arithmetic progression	B1	1	Any mention of arithmetic
-----				
	(iv) $\frac{2N}{2}(16 + (2N - 1)3) - \frac{N}{2}(16 + (N - 1)3) = 1256$  $26N + 12N^2 - 13N - 3N^2 = 2512$ $9N^2 + 13N - 2512 = 0$  $(9N + 157)(N - 16) = 0$ $N = 16$	M1  M1 M1*  M1dep* A1	5	Attempt $S_N$ , using any correct formula (inc $\sum (3n + 5)$ ) Attempt $S_{2N}$ , using any correct formula, with $2N$ consistent (inc $\sum (3n + 5)$ ) Attempt subtraction (correct order) and equate to 1256 Attempt to solve quadratic in $N$ Obtain $N = 16$ only, from correct working
				OR: alternative method is to use $\frac{n}{2}(a + l) = 1256$ M1 Attempt given difference as single summation with $N$ terms M1 Attempt $a = u_{N+1}$ M1 Attempt $l = u_{2N}$ M1 Equate to 1256 and attempt to solve quadratic A1 Obtain $N = 16$ only, from correct working
				<b>10</b>

9 (i)



M1 Reasonable graph in both quadrants  
 A1 Correct graph in both quadrants

B1 3 State or imply (0, 6)

(ii)  $9^x = 150$ 

$$x \log 9 = \log 150$$

$$x = 2.28$$

M1 Introduce logarithms throughout, or equiv with  $\log_9$

M1 Use  $\log a^b = b \log a$  and attempt correct method to find  $x$

A1 3 Obtain  $x = 2.28$

(iii)  $6 \times 5^x = 9^x$ 

$$\log_3 (6 \times 5^x) = \log_3 9^x$$

$$\log_3 6 + x \log_3 5 = x \log_3 9$$

$$\log_3 3 + \log_3 2 + x \log_3 5 = 2x$$

$$x(2 - \log_3 5) = 1 + \log_3 2$$

$$x = \frac{1 + \log_3 2}{2 - \log_3 5} \quad \mathbf{A.G.}$$

M1 Form eqn in  $x$  and take logs throughout (any base)

M1 Use  $\log a^b = b \log a$  correctly on  $\log 5^x$  or  $\log 9^x$  or legitimate combination of these two

M1 Use  $\log ab = \log a + \log b$  correctly on  $\log (6 \times 5^x)$  or  $\log 6$

M1 Use  $\log_3 9 = 2$  or equiv (need base 3 throughout that line)

A1 5 Obtain  $x = \frac{1 + \log_3 2}{2 - \log_3 5}$  convincingly  
 (inc base 3 throughout)

11
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## 4723 Core Mathematics 3

- 1 Obtain integral of form  $k(2x-7)^{-1}$  M1 any constant  $k$   
 Obtain correct  $-5(2x-7)^{-1}$  A1 or equiv  
 Include ... +  $c$  B1 **3** at least once; following any integral  
**3**

- 2 (i) Use  $\sin 2\theta = 2\sin\theta\cos\theta$  B1  
 Attempt value of  $\sin\theta$  from  $k\sin\theta\cos\theta = 5\cos\theta$  M1 any constant  $k$ ; or equiv  
 Obtain  $\frac{5}{12}$  A1 **3** or exact equiv; ignore subsequent work

- (ii) Use  $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$  or  $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$  B1 or equiv  
 Attempt to produce equation involving  $\cos\theta$  only M1 using  $\sin^2\theta = \pm 1 \pm \cos^2\theta$  or equiv  
 Obtain  $3\cos^2\theta + 8\cos\theta - 3 = 0$  A1 or equiv  
 Attempt solution of 3-term quadratic equation M1 using formula or factorisation or equiv  
 Obtain  $\frac{1}{3}$  as only final value of  $\cos\theta$  A1 **5** or exact equiv; ignore subsequent work  
**8**

- 3 (i) Obtain or clearly imply  $60\ln x$  B1  
 Obtain  $(60\ln 20 - 60\ln 10)$  and hence  $60\ln 2$  B1 **2** with no error seen
- (ii) Attempt calculation of form  $k(y_0 + 4y_1 + y_2)$  M1 any constant  $k$ ; using  $y$ -value attempts  
 Identify  $k$  as  $\frac{5}{3}$  A1  
 Obtain  $\frac{5}{3}(6 + 4 \times 4 + 3)$  and hence  $\frac{125}{3}$  or 41.7 A1 **3** or equiv
- (iii) Equate answers to parts (i) and (ii) M1 provided  $\ln 2$  involved  
 Obtain  $60\ln 2 = \frac{125}{3}$  and hence  $\frac{25}{36}$  A1 **2** AG; necessary detail required including clear use of an exact value from (ii)  
**7**

- 4 (i) Attempt correct process for composition M1 numerical or algebraic  
 Obtain  $(7$  and hence)  $0$  A1 **2**
- (ii) Attempt to find  $x$ -intercept M1  
 Obtain  $x \leq 7$  A1 **2** or equiv; condone use of  $<$
- (iii) Attempt correct process for finding inverse M1  
 Obtain  $\pm(2-y)^3 - 1$  or  $\pm(2-x)^3 - 1$  A1  
 Obtain correct  $(2-x)^3 - 1$  A1 **3** or equiv in terms of  $x$
- (iv) Refer to reflection in  $y = x$  B1 **1** or clear equiv  
**8**

5 (i)	Obtain derivative of form $kx(x^2 + 1)^7$ Obtain $16x(x^2 + 1)^7$ Equate first derivative to 0 and confirm $x = 0$ or substitute $x = 0$ and verify first derivative zero Refer, in some way, to $x^2 + 1 = 0$ having no root	M1 any constant $k$ A1 or equiv M1 AG; allow for deriv of form $kx(x^2 + 1)^7$ A1 <b>4</b> or equiv
-----		
(ii)	Attempt use of product rule Obtain $16(x^2 + 1)^7 + \dots$ Obtain $\dots + 224x^2(x^2 + 1)^6$  Substitute 0 in attempt at second derivative Obtain 16	*M1 obtaining $\dots + \dots$ form A1√ follow their $kx(x^2 + 1)^7$ A1√ follow their $kx(x^2 + 1)^7$ ; or unsimplified equiv M1 dep *M A1 <b>5</b> from second derivative which is correct at some point
<b>9</b>		
-----		
6	Integrate $e^{3x}$ to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$ Obtain indefinite integral of form $m_1e^{3x} + m_2e^{-\frac{1}{2}x}$ Obtain correct $\frac{1}{3}ke^{3x} - 2(k-2)e^{-\frac{1}{2}x}$  Obtain $e^{3\ln 4} = 64$ or $e^{-\frac{1}{2}\ln 4} = \frac{1}{2}$ Apply limits and equate to 185 Obtain $\frac{64}{3}k - (k-2) - \frac{1}{3}k + 2(k-2) = 185$ Obtain $\frac{17}{2}$	B1 or both M1 any constants $m_1$ and $m_2$ A1 or equiv  B1 or both M1 including substitution of lower limit A1 or equiv A1 <b>7</b> or equiv
<b>7</b>		
-----		
7 (a)	<u>Either</u> : State or imply either $\frac{dA}{dr} = 2\pi r$ or $\frac{dA}{dt} = 250$ Attempt manipulation of derivatives to find $\frac{dr}{dt}$  Obtain correct $\frac{250}{2\pi r}$ Obtain 1.6  <u>Or</u> : Attempt to express $r$ in terms of $t$ Obtain $r = \sqrt{\frac{250t}{\pi}}$ Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$ Substitute $t = 7.6$ to obtain 1.6	B1 or both  M1 using multiplication / division A1 or equiv A1 <b>4</b> or equiv; allow greater accuracy  M1 using $A = 250t$ A1 or equiv M1 any constant $k$ A1 ( <b>4</b> ) allow greater accuracy
-----		

- (b) State  $\frac{dm}{dt} = -150ke^{-kt}$  B1  
 Equate to  $(\pm)3$  and attempt value for  $t$  M1 using valid process; condone sign confusion  
 Obtain  $-\frac{1}{k}\ln\left(\frac{1}{50k}\right)$  or  $\frac{1}{k}\ln(50k)$  or  $\frac{\ln 50 + \ln k}{k}$  A1 **3** or equiv but with correct treatment of signs  
7

- 8 (i) State scale factor is  $\sqrt{2}$  B1 allow 1.4  
 State translation is in negative  $x$ -direction ... B1 or clear equiv  
 ... by  $\frac{3}{2}$  units B1 **3**
- 
- (ii) Draw (more or less) correct sketch of  $y = \sqrt{2x+3}$  B1 'starting' at point on negative  $x$ -axis  
 Draw (more or less) correct sketch of  $y = \frac{N}{x^3}$  B1 showing both branches  
 Indicate one point of intersection B1 **3** with both sketches correct  
 [SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]
- 
- (iii) (a) Substitute 1.9037 into  $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$  M1 or into equation  $\sqrt{2x+3} = \frac{N}{x^3}$ ; or equiv  
 Obtain 18 or value rounding to 18 A1 **2** with no error seen
- 
- (b) State or imply  $2.6282 = N^{\frac{1}{3}}(2 \times 2.6022 + 3)^{-\frac{1}{6}}$  B1  
 Attempt solution for  $N$  M1 using correct process  
 Obtain 52 A1 **3** concluding with integer value  
11

- 9 (i) Identify  $\tan 55^\circ$  as  $\tan(45^\circ + 10^\circ)$  B1 or equiv  
 Use correct angle sum formula for  $\tan(A+B)$  M1 or equiv  
 Obtain  $\frac{1+p}{1-p}$  A1 **3** with  $\tan 45^\circ$  replaced by 1
- 
- (ii) Either: Attempt use of identity for  $\tan 2A$  \*M1 linking  $10^\circ$  and  $5^\circ$   
 Obtain  $p = \frac{2t}{1-t^2}$  A1  
 Attempt solution for  $t$  of quadratic equation M1 dep \*M  
 Obtain  $\frac{-1 + \sqrt{1+p^2}}{p}$  A1 **4** or equiv; and no second expression
- Or (1): Attempt expansion of  $\tan(60^\circ - 55^\circ)$  \*M1  
 Obtain  $\frac{\sqrt{3} - \frac{1+p}{1-p}}{1 + \sqrt{3} \frac{1+p}{1-p}}$  A1√ follow their answer from (i)  
 Attempt simplification to remove denominators M1 dep \*M  
 Obtain  $\frac{\sqrt{3}(1-p) - (1+p)}{1-p + \sqrt{3}(1+p)}$  A1 **(4)** or equiv

<u>Or (2):</u> State or imply $\tan 15^\circ = 2 - \sqrt{3}$	B1
Attempt expansion of $\tan(15^\circ - 10^\circ)$	M1 with exact attempt for $\tan 15^\circ$
Obtain $\frac{2 - \sqrt{3} - p}{1 + p(2 - \sqrt{3})}$	A2 (4)
<u>Or (3):</u> State or imply $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1 or exact equiv
Attempt expansion of $\tan(15^\circ - 10^\circ)$	M1 with exact attempt for $\tan 15^\circ$
Obtain $\frac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 (4) or equiv
<u>Or (4):</u> Attempt expansion of $\tan(10^\circ - 5^\circ)$	*M1
Obtain $t = \frac{p-t}{1+pt}$	A1
Attempt solution for $t$ of quadratic equation	M1 dep *M
Obtain $\frac{-2 + \sqrt{4+4p^2}}{2p}$	A1 (4) or equiv; and no second expression
-----	
(iii) Attempt expansion of both sides	M1
Obtain $3\sin\theta\cos 10^\circ + 3\cos\theta\sin 10^\circ =$ $7\cos\theta\cos 10^\circ + 7\sin\theta\sin 10^\circ$	A1 or equiv
Attempt division throughout by $\cos\theta\cos 10^\circ$	M1 or by $\cos\theta$ (or $\cos 10^\circ$ ) only
Obtain $3t + 3p = 7 + 7pt$	A1 or equiv
Obtain $\frac{3p-7}{7p-3}$	A1 5 or equiv

12
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# 4724 Core Mathematics 4

## 1 Long division method

Correct leading term $x^2$ in quotient	B1	
Evidence of correct div process	M1	Sufficient to convince
(Quotient =) $x^2 + 6x - 4$	A1	
(Remainder =) $11x + 9$	A1	

## Identity method

$x^4 + 11x^3 + 28x^2 + 3x + 1 = Q(x^2 + 5x + 2) + R$	M1	
$Q = ax^2 + bx + c$ or $x^2 + bx + c$ ; $R = dx + e$ & $\geq 3$ ops	M1	N.B. $a = 1 \Rightarrow 1$ of the 3 ops
$a = 1, b = 6, c = -4, d = 11, e = 9$ (for all 5)	A2	S.R. <u>B1</u> for 3 of these

**4**

2 (i) Find at least 2 of $(\vec{AB}$ or $\vec{BA}), (\vec{BC}$ or $\vec{CB}), (\vec{AC}$ or $\vec{CA})$	M1	irrespective of label; any notation
Use correct method to find scal prod of any 2 vectors	M1	<u>or</u> use corr meth for modulus
Use $\vec{AB} \cdot \vec{BC} = 0$ or $\frac{\vec{AB} \cdot \vec{BC}}{ \vec{AB}  \vec{BC} } = 0$	M1	or use $ \vec{AB} ^2 +  \vec{BC} ^2 =  \vec{AC} ^2$
Obtain $p = 1$ (dep 3 @ M1)	A1	<b>4</b>

(ii) Use equal ratios of appropriate vectors	M1	or scalar product method
Obtain $p = -8$	A1	<b>2</b>

**6**

3 Use $\cos 2x = a \cos^2 x + b / \pm \cos^2 x - \sin^2 x / 1 - 2\sin^2 x$	*M1	
Obtain $\lambda + \mu \sec^2 x$	dep*M1	using 'reasonable' Pythag attempt
$\int \lambda + \mu \sec^2 x \, dx = \lambda x + \mu \tan x$	A1	( $\lambda$ or $\mu$ may be 0 here/prev line)
Obtain correct result $2x - \tan x$	A1	no follow-through
$\frac{1}{6}\pi - \sqrt{3} + 1$ ISW	A1	exact answer required

**5**

4 Attempt to connect $du$ and $dt$ or find $\frac{du}{dt}$ or $\frac{dt}{du}$	M1	not $du = dt$ but no accuracy
$du = \frac{1}{t} dt$ or $\frac{du}{dt} = \frac{1}{t}$ or $dt = e^{u-2} du$ or $\frac{dt}{du} = e^{u-2}$	A1	
Indef int $\rightarrow \int \frac{1}{u^2} (du)$	A1	no $t$ or $dt$ in evidence
$= -\frac{1}{u}$	A1	
Attempt to change limits if working with $f(u)$	M1	or re-subst & use 1 and e
$\frac{1}{6}$ ISW	A1	ln e must be changed to 1, ln 1 to 0

**6**



5	(i) $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \dots$ $\dots - \frac{1}{9}x^2$	B1 B1 2	 $-\frac{2}{18}x^2$ acceptable
-----			
(ii)	(a) $(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$ $(1+2x)^{\frac{1}{3}} =$ their (i) expansion with $2x$ replacing $x$ $= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \dots$ Required expansion = 2 (expansion just found)	B1 M1 $\sqrt{A1}$ $\sqrt{B1}$ 4	not $16^{\frac{1}{3}}(\frac{1}{2}+x)^{\frac{1}{3}}$ not dep on prev B1 $-\frac{8}{18}x^2$ acceptable accept equiv fractions
<b>N.B.</b> If not based on part (i), award M1 for $8^{\frac{1}{3}} + \frac{1}{3} \cdot 8^{-\frac{2}{3}}(16x) + \frac{\frac{1}{3} \cdot -\frac{2}{3}}{1.2} 8^{-\frac{5}{3}}(16x)^2$ , allowing $16x^2$ for $(16x)^2$ , with 3 @ A1 for $2\dots + \frac{4}{3}x\dots - \frac{8}{9}x^2$ , accepting equivalent fractions & ISW			
(ii)	(b) $-\frac{1}{2} < x < \frac{1}{2}$ or $ x  < \frac{1}{2}$	B1 1	no equality
<b>7</b>			
6	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW $\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW Stating/implying $\frac{3t^2 - \frac{3}{t}}{9 - \frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$ $t = 3$ as final ans with clear log indication of invalidity of $-3$ ; ignore (non) mention of $t = 0$	M1 B1 B1 A1 A2	quoted/implied  WWW, totally correct at this stage <b>S.R.</b> A1 if $t = \pm 3$ or $t = -3$ or ( $t = 3$ & wrong/no indication)
<b>6</b>			
7	Treat $\frac{d}{dx}(x^2y)$ as a product $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ $3x^2 + 2x^2 \frac{dy}{dx} + 4xy = 3y^2 \frac{dy}{dx}$ Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa $\frac{dy}{dx} = -4$ WWW grad normal = $-\frac{1}{\text{their } \frac{dy}{dx}}$ Find eqn of line, through (2, 1), with either gradient $x - 4y + 2 = 0$	M1 B1 A1 M1 A1 $\sqrt{A1}$ M1 A1	Ignore $\frac{dy}{dx} =$ if not used  stated or used using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their } \frac{dy}{dx}}$ AEF with integral coefficients
<b>8</b>			

<b>8 (i)</b>	$-\sin x e^{\cos x}$	B1	<b>1</b>
<b>(ii)</b>	$\int \sin x e^{\cos x} dx = -e^{\cos x}$	B1	anywhere in part (ii)
	Parts with split $u = \cos x, dv = \sin x e^{\cos x}$	M1	result $f(x) +/ - \int g(x) dx$
	Indef Integ, 1st stage $-\cos x e^{\cos x} - \int \sin x e^{\cos x} dx$	A1	accept ... $-\int -e^{\cos x} \cdot -\sin x dx$
	Second stage = $-\cos x e^{\cos x} + e^{\cos x}$	*A1	
	Final answer = 1	dep*A2	<b>6</b>

<b>7</b>
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<b>9 (i)</b>	$P$ is $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$	B1	
	direction vector of $\ell$ is $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and of $\overrightarrow{OP}$ is their $P$	$\sqrt{B1}$	
	Use $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}   \mathbf{b} }$ for $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and their OP	M1	
	$\theta = 35.3$ or better (0.615... rad)	A1	<b>4</b>

<b>(ii)</b>	Use $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix} = 0$	M1	
	$1(3+t) - 1(1-t) + 2(1+2t) = 0$	A1	
	$t = -\frac{2}{3}$	A1	
	Subst. into $\begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix}$ to produce $\begin{pmatrix} 7/3 \\ 5/3 \\ -1/3 \end{pmatrix}$ ISW	A1	<b>4</b>

<b>(iii)</b>	Use $\sqrt{x^2 + y^2 + z^2}$ where $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is part (ii) answer	M1	
	Obtain $\sqrt{\frac{75}{9}}$ AEF, 2.89 or better (2.8867513...)	A1	<b>2</b>

<b>10</b>
-----------

10 (i)  $\frac{\frac{1}{3}}{3-x} \dots\dots\dots -\frac{\frac{1}{3}}{6-x}$  B1+1 2

(ii) (a) Separate variables  $\int \frac{1}{(3-x)(6-x)} dx = \int k dt$  M1 or invert both sides

Style: For the M1, dx & dt must appear on correct sides or there must be  $\int$  sign on both sides

Change  $\frac{1}{(3-x)(6-x)}$  into partial fractions from (i)  $\sqrt{B1}$

$\int \frac{A}{3-x} dx = \left(-A \text{ or } -\frac{1}{A}\right) \ln(3-x)$  B1 or  $\int \frac{B}{6-x} dx = \left(-B \text{ or } -\frac{1}{B}\right) \ln(6-x)$

$-\frac{1}{3} \ln(3-x) + \frac{1}{3} \ln(6-x) = kt (+c)$   $\sqrt{A1}$  f.t. from wrong multiples in (i)

Subst  $(x = 0, t = 0)$  &  $(x = 1, t = 1)$  into eqn with 'c' M1 and solve for 'k'

Use  $\ln a + \ln b = \ln ab$  or  $\ln a - \ln b = \ln \frac{a}{b}$  M1

Obtain  $k = \frac{1}{3} \ln \frac{5}{4}$  with sufficient working & WWW A1 7 AG

(b) Substitute  $k = \frac{1}{3} \ln \frac{5}{4}$ ,  $t = 2$  & their value of 'c' \*M1

Reduce to an eqn of form  $\frac{6-x}{3-x} = \lambda$  dep\*M1 where  $\lambda$  is a const

Obtain  $x = \frac{27}{17}$  or 1.6 or better (1.5882353...) A2 4 S.R. A1  $\sqrt{}$  for  $x = \frac{3\lambda - 6}{\lambda - 1}$

**13**

# 4725 Further Pure Mathematics 1

1 (i)	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1	Two elements correct
		B1	2 Remaining elements correct
<hr style="border-top: 1px dashed black;"/>			
(ii)	$4a - 6$	B1	Correct determinant
		M1	Equate det A to 0 and solve
	$a = \frac{3}{2}$	A1	3 Obtain correct answer a. e. f.
		<b>5</b>	
<hr/>			
2 (i)	$u^3 - 3u^2 + 3u - 1$	B1	Correct unsimplified expansion of $(u-1)^3$
		M1	Substitute for $x$
	$2u^3 - 6u^2 + 9u - 8 = 0$	A1	3 Obtain correct <b>equation</b>
<hr style="border-top: 1px dashed black;"/>			
(ii)		M1	Use $(\pm)\frac{d}{a}$ of new equation
	4	A1ft	2 Obtain correct answer from their equation
		<b>5</b>	
<hr/>			
3	$x - iy$	B1	Conjugate known
		M1	Equate real and imaginary parts
	$x + 2y = 12 \quad 2x + y = 9$	A1	Obtain both equations, OK with factor of $i$
		M1	Solve pair of equations
	$z = 2 + 5i$	A1	5 Obtain correct answer as a complex number
			S.C. Solving $z + 2iz = 12 + 9i$ can get max $4/5$ , not first B1
		<b>5</b>	
<hr/>			
4		M1	Express as sum of three series
		M1	Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1	Obtain correct unsimplified answer
		M1	Attempt to factorise
		A1	Obtain at least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	6 Obtain fully factorised correct answer
		<b>6</b>	

5 (i)	B1 B1	2	Rotation 90° (about origin) Anticlockwise
<hr/>			
(ii) <i>Either</i>	M1		Show image of unit square after reflection in $y = -x$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1		Deduce reflection in $x$ -axis
<i>Or</i>	B1ft B1ft M1	4	Each column correct ft for matrix of their transformation Post multiply by correct reflection matrix
	A1 B1B1		Obtain correct answer State reflection, in $x$ -axis
			<b>S.C.</b> If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
			<b>6</b>
<hr/>			
6 (i)	B1 M1		State or use $5 + i$ as a root Use $\sum \alpha\beta = 6$
$x = -2$	A1	3	Obtain correct answer
<hr/>			
(ii) <i>Either</i>	M1		Use $p = -\sum \alpha$
$p = -8$	A1ft M1		Obtain correct answer, from their root Use $q = -\alpha\beta\gamma$
$q = 52$	A1ft	4	Obtain correct answer, from their root
<i>Or</i>	M1 M1 A1A1		Attempt to find quadratic factor Attempt to expand quadratic and linear Obtain correct answers
<i>Or</i>	M1 M1 A1 A1ft		Substitute $(5 - i)$ into equation Equate real and imaginary parts Obtain correct answer for $p$ Obtain correct answer for $q$ , ft their $p$
			<b>7</b>
<hr/>			
7 (i)	B1	1	Obtain <b>given</b> answer correctly
<hr/>			
(ii)	M1		Express at least 1 <sup>st</sup> two and last term using (i)
	A1		All terms correct
	M1		Show that correct terms cancel
$1 - \frac{1}{(n+1)^2}$	A1	4	Obtain correct answer, in terms of $n$
<hr/>			
(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
	B1	2	Obtain correct answer <b>S.C.</b> $-\frac{3}{4}$ scores B1
			<b>7</b>

<p>8 (i)</p> $x^2 - y^2 = 5 \text{ and } xy = -6$ $\pm(3 - 2i)$	<p>M1 Attempt to equate real and imaginary parts of <math>(x + iy)^2</math> &amp; <math>5 - 12i</math></p> <p>A1 Obtain both results, a.e.f</p> <p>M1 Obtain quadratic in <math>x^2</math> or <math>y^2</math></p> <p>M1 Solve to obtain <math>x = (\pm)3</math> or <math>y = (\pm)2</math></p> <p>A1 <b>5</b> Obtain correct answers as complex nos</p>
-----	
<p>(ii)</p> <p>square root</p>	<p>B1ft Circle with centre at their</p> <p>B1 Circle passing through origin</p> <p>B1ft 2<sup>nd</sup> circle centre correct relative to 1<sup>st</sup></p> <p>B1 <b>4</b> Circle passing through origin</p> <p><b>9</b></p>
-----	
<p>9 (i)</p> $\det \mathbf{A} = \Delta = 6a - 6$ $\mathbf{A}^{-1} = \frac{1}{\Delta} \begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	<p>M1 Show correct expansion process for <math>3 \times 3</math> or multiply adjoint by <math>\mathbf{A}</math></p> <p>M1 Correct evaluation of any <math>2 \times 2</math> at any stage</p> <p>A1 Obtain correct answer</p> <p>M1 Show correct process for adjoint entries</p> <p>A1 Obtain at least 4 correct entries in adjoint</p> <p>B1 Divide by their determinant</p> <p>A1 <b>7</b> Obtain completely correct answer</p>
-----	
<p>(ii)</p> $\frac{1}{\Delta} \begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	<p>M1 Attempt product of form <math>\mathbf{A}^{-1}\mathbf{C}</math> or eliminate to get 2 equations and solve</p> <p>A1A1A1 Obtain correct answer</p> <p>ft all 3</p> <p><b>4</b> S.C. if det now omitted, allow max A2 ft</p> <p><b>11</b></p>
-----	
<p>10 (i)</p> $\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \quad \mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	<p>B1 Correct <math>\mathbf{M}^2</math> seen</p> <p>M1 Convincing attempt at matrix multiplication for <math>\mathbf{M}^3</math></p> <p>A1 <b>3</b> Obtain correct answer</p>
-----	
<p>(ii) <math>\mathbf{M}^n = \begin{pmatrix} 1 &amp; 2n \\ 0 &amp; 1 \end{pmatrix}</math></p>	<p>B1ft <b>1</b> State correct form, consistent with (i)</p>
-----	

10 (iii)

M1		Correct attempt to multiply $\mathbf{M}$ & $\mathbf{M}^k$ or v.v.
A1		Obtain element $2(k+1)$
A1		Clear statement of induction step, from correct working
B1	<b>4</b>	Clear statement of induction conclusion, following their working

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 (iv)

B1		Shear
DB1		$x$ -axis invariant
DB1	<b>3</b>	e.g. $(1, 1) \rightarrow (21, 1)$ or equivalent using scale factor or angles

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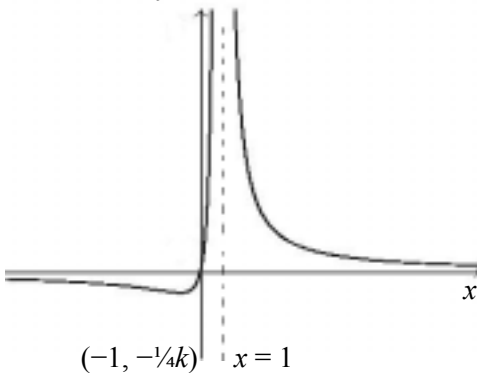
<b>11</b>
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## 4726 Further Pure Mathematics 2

1	(i)	Get 0.876096, 0.876496, 0.876642	B1√	For any one correct or √ from wrong answer; radians only
			B1	All correct
2	(ii)	Subtract correctly (0.00023(0), 0.000084)	B1√	On their answers
		Divide their errors as $e_4/e_3$ only	M1	May be implied
		Get 0.365(21...)	A1	Cao
3	(i)	Find $f'(x) = 1/(1+(1+x)^2)$	M1	Quoted or derived; may be simplified or left as $\sec^2 y \, dy/dx = 1$
		Get $f(0) = 1/4\pi$ and $f'(0) = 1/2$	A1√	On their $f'(0)$ ; allow $f(0)=0.785$ but not 45
		Attempt $f''(x)$	M1	Reasonable attempt at chain/quotient rule or implicit differentiation
		Correctly get $f''(0) = -1/2$	A1	A.G.
3	(ii)	Attempt Maclaurin as $af(0)+bf'(0)+cf''(0)$	M1	Using their $f(0)$ and $f'(0)$
		Get $1/4\pi + 1/2x - 1/4x^2$	A1	Cao; allow 0.785
4	(i)	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$	M1	Allow reasonable $y$ -step/ $x$ -step
		Equate to gradient of curve at $x_1$	M1	Allow $\pm$
		Clearly arrive at A.G.	A1	Beware confusing use of $\pm$
		SC Attempt equation of tangent	M1	As $y - f(x_1) = f'(x_1)(x - x_1)$
		Put $(x_2, 0)$ into their equation	M1	
4	(ii)	Clearly arrive at A.G.	A1	
		Diagram showing at least one more tangent	B1	
		Description of tangent meeting $x$ -axis, used as next starting value	B1	
4	(iii)	Reasonable attempt at N-R	M1	Clear attempt at differentiation
		Get 1.60	A1	Or answer which rounds
4	(i)	State $r=1$ and $\theta=0$ .	B1	May be seen or implied
			B1	Correct shape, decreasing $r$ (not through $O$ )
4	(ii)	Use $1/2 \int r^2 \, d\theta$ with $r = e^{-2\theta}$ seen or implied	M1	Allow $1/2 \int e^{4\theta} \, d\theta$
		Integrate correctly as $-1/8 e^{-4\theta}$	A1	
		Use limits in correct order	M1	In their answer
		Use $r_1^2 = e^{-4\theta}$ etc.	M1	May be implied
		Clearly get $k=1/8$	A1	



<b>5</b>	<b>(i)</b>	Use correct definitions of cosh and sinh	B1	
		Attempt to square and subtract	M1	On their definitions
		Clearly get A.G.	A1	
		Show division by $\cosh^2$	B1	Or clear use of first result
<hr/>				
	<b>(ii)</b>	Rewrite as quadratic in sech and attempt to solve	M1	Or quadratic in cosh
		Eliminate values outside $0 < \operatorname{sech} \leq 1$	B1	Or eliminate values outside $\cosh \geq 1$ (allow positive)
		Get $x = \ln(2+\sqrt{3})$	A1	
		Get $x = -\ln(2+\sqrt{3})$ or $\ln(2-\sqrt{3})$	A1	
<hr/>				
<b>6</b>	<b>(i)</b>	Attempt at correct form of P.F.	M1	Allow $Cx/(x^2+1)$ here; not $C = 0$
		Rewrite as $4 =$		
		$A(1+x)(1+x^2) + B(1-x)(1+x^2) + (Cx+D)(1-x)(1+x)$	M1	From their P.F.
		Use values of $x$ /equate coefficients	M1	
		Get $A = 1, B = 1$	A1	cwo
		Get $C = 0, D = 2$	A1	
				SC Use of cover-up rule for $A, B$ M1 If both correct A1 cwo
<hr/>				
	<b>(ii)</b>	Get $A \ln(1+x) - B \ln(1-x)$	M1	Or quote from List of Formulae
		Get $D \tan^{-1}x$	B1	
		Use limits in their integrated expressions	M1	
		Clearly get A.G.	A1	
<hr/>				
<b>7</b>	<b>(i)</b>	LHS = sum of areas of rectangles, area = $1 \times y$ -value from $x = 1$ to $x = n$	B1	
		RHS = Area under curve from $x = 0$ to $n$	B1	
<hr/>				
	<b>(ii)</b>	Diagram showing areas required	B1	
		Use sum of areas of rectangles	B1	
		Explain/show area inequality with limits in integral clearly specified	B1	
<hr/>				
	<b>(iii)</b>	Attempt integral as $kx^{4/3}$	M1	
		Limits gives 348(.1) and 352(.0)	A1	Allow one correct
		Get 350	A1	From two correct values only

8	(i)	Get $x = 1, y = 0$	B1, B1	
	(ii)	Rewrite as quadratic in $x$ Use $b^2 - 4ac \geq 0$ for all real $x$ Get correct inequality State use of $k > 0$ to A.G.	M1 M1 A1 A1	$(x^2y - x(2y + k) + y = 0)$ Allow $>, =$ here $4ky + k^2 \geq 0$
				SC Use differentiation (parts (ii) and (iii)) Attempt prod/quotient rule M1 Solve $= 0$ for $x = -1$ A1 Use $x = -1$ only (reject $x = 1$ ), $y = -1/4k$ A1 Fully justify minimum B1 Attempt to justify for all $x$ M1 Clearly get A.G. A1
	(iii)	Replace $y = -1/4k$ in quadratic in $x$ Get $x = -1$ only	M1 A1	
			B1 B1	Through origin with minimum at $(-1, -1/4k)$ seen or given in the answer Correct shape (asymptotes and approaches)
				SC (Start again) Differentiate and solve $dy/dx = 0$ for at least one $x$ -value, independent of $k$ M1 Get $x = -1$ only A1
9	(i)	Rewrite $\tanh y$ as $(e^y - e^{-y})/(e^y + e^{-y})$ Attempt to write as quadratic in $e^{2y}$ Clearly get A.G.	B1 M1 A1	Or equivalent
	(ii)	(a)	M1 A1 B1	
		Attempt to diff. and solve $= 0$ Get $\tanh x = b/a$ Use $(-1) < \tanh x < 1$ to show $b < a$		SC Use exponentials M1 Get $e^{2x} = (a + b)/(a - b)$ A1 Use $e^{2x} > 0$ to show $b < a$ B1
				SC Write $x = \tanh^{-1}(b/a)$ M1 $= 1/2 \ln((1 + b/a)/(1 - b/a))$ A1 Use $( ) > 0$ to show $b < a$ B1
		(b)	B1 M1 A1 M1 A1 B1	
		Get $\tanh x = 1/a$ from part (ii)(a) Replace as $\ln$ from their answer Get $x = 1/2 \ln((a + 1)/(a - 1))$ Use $e^{1/2 \ln((a+1)/(a-1))} = \sqrt{(a + 1)/(a - 1)}$ Clearly get A.G. Test for minimum correctly		At least once
				SC Use of $y = \cosh x(a - \tanh x)$ and $\cosh x = 1/\operatorname{sech} x = 1/\sqrt{1 - \tanh^2 x}$

## 4727 Further Pure Mathematics 3

<b>1</b>	<b>METHOD 1</b>		
	line segment between $l_1$ and $l_2 = \pm[4, -3, -9]$	B1	For correct vector
	$\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$	M1* A1	For finding vector product of direction vectors
	distance = $\frac{ [4, -3, -9] \cdot [-2, 0, 1] }{\sqrt{2^2 + 0^2 + 1^2}} = \frac{17}{\sqrt{5}}$	M1 (*dep)	For using numerator of distance formula
	$\neq 0$ , so skew	A1 <b>5</b>	For correct scalar product and correct conclusion
<hr/>			
	<b>METHOD 2</b> lines would intersect where		
	$\begin{cases} 1 + s = -3 + 2t \\ -2 - s = 1 + 3t \\ -4 + 2s = 5 + 4t \end{cases} \Rightarrow \begin{cases} s - 2t = -4 \\ s + 3t = -3 \\ 2s - 4t = 9 \end{cases}$	B1 M1* A1	For correct parametric form for either line For 3 equations using 2 different parameters
	$\Rightarrow$ contradiction, so skew	M1 (*dep) A1	For attempting to solve to show (in)consistency For correct conclusion
<hr/>			
<b>5</b>			
<b>2 (i)</b>	$(a + b\sqrt{5})(c + d\sqrt{5})$ $= ac + 5bd + (bc + ad)\sqrt{5} \in H$	M1 A1 <b>2</b>	For using product of 2 distinct elements For correct expression
<b>(ii)</b>	$(e =) 1 \text{ OR } 1 + 0\sqrt{5}$	B1 <b>1</b>	For correct identity
<b>(iii)</b>	<i>EITHER</i> $\frac{1}{a + b\sqrt{5}} \times \frac{a - b\sqrt{5}}{a - b\sqrt{5}}$ <i>OR</i> $(a + b\sqrt{5})(c + d\sqrt{5}) = 1 \Rightarrow \begin{cases} ac + 5bd = 1 \\ bc + ad = 0 \end{cases}$ inverse = $\frac{a}{a^2 - 5b^2} - \frac{b}{a^2 - 5b^2}\sqrt{5}$	M1 A1 <b>2</b>	For correct inverse as $(a + b\sqrt{5})^{-1}$ and multiplying top and bottom by $a - b\sqrt{5}$ <i>OR</i> for using definition and equating parts For correct inverse. Allow as a single fraction
<b>(iv)</b>	5 is prime <i>OR</i> $\sqrt{5} \notin \mathbb{Q}$	B1 <b>1</b>	For a correct property (or equivalent)
<hr/>			
<b>6</b>			
<b>3</b>	Integrating factor = $e^{\int 2dx} = e^{2x}$ $\Rightarrow \frac{d}{dx}(ye^{2x}) = e^{-x}$ $\Rightarrow ye^{2x} = -e^{-x} + c$ $(0, 1) \Rightarrow c = 2$ $\Rightarrow y = -e^{-3x} + 2e^{-2x}$	B1 M1 A1 M1 A1√ A1 <b>6</b>	For correct IF For $\frac{d}{dx}(y \cdot \text{their IF}) = e^{-3x}$ . their IF For correct integration both sides For substituting (0, 1) into their GS and solving for $c$ For correct $c$ f.t. from their GS For correct solution
<hr/>			
<b>6</b>			
<b>4 (i)</b>	$(z =) 2, -2, 2i, -2i$	M1 A1 <b>2</b>	For at least 2 roots of the form $k\{1, i\}$ <b>AEF</b> For correct values

<p>(ii) <math>\frac{w}{1-w} = 2, -2, 2i, -2i</math></p> $w = \frac{z}{1+z}$ $w = \frac{2}{3}, 2$ $w = \frac{4}{5} \pm \frac{2}{5}i$	<p>M1 M1 B1 A1 A1</p>	<p>For <math>\frac{w}{1-w} =</math> any one solution from (i)</p> <p>For attempting to solve for <math>w</math>, using any solution or in general</p> <p>For any one of the 4 solutions</p> <p>For both real solutions</p> <p>For both complex solutions</p> <p><b>SR</b> Allow B1√ and one A1√ from <math>k \neq 2</math></p>
<b>7</b>		
<p>5 (i) <math>\mathbf{AB} = k\left[\frac{2}{3}\sqrt{3}, 0, -\frac{2}{3}\sqrt{6}\right]</math>,  <math>\mathbf{BC} = k\left[-\sqrt{3}, 1, 0\right]</math>, <math>\mathbf{CA} = k\left[\frac{1}{3}\sqrt{3}, -1, \frac{2}{3}\sqrt{6}\right]</math>  <math>\mathbf{n} = k_1\left[\frac{2}{3}\sqrt{6}, \frac{2}{3}\sqrt{18}, \frac{2}{3}\sqrt{3}\right] = k_2\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right]</math>  substitute <math>A, B</math> or <math>C \Rightarrow x + \sqrt{3}y + \frac{1}{2}\sqrt{2}z = \frac{2}{3}\sqrt{3}</math></p>	<p>B1 B1 M1 M1 A1</p>	<p>For any one edge vector of <math>\triangle ABC</math></p> <p>For any other edge vector of <math>\triangle ABC</math></p> <p>For attempting to find vector product of any two edges</p> <p>For substituting <math>A, B</math> or <math>C</math> into <math>\mathbf{r} \cdot \mathbf{n}</math></p> <p>For correct equation <b>AG</b></p> <p><b>SR</b> For verification only allow M1, then A1 for 2 points and A1 for the third point</p>
<p>(ii) Symmetry in plane <math>OAB</math> or <math>Oxz</math> or <math>y = 0</math></p>	<p>B1* B1 (*dep)2</p>	<p>For quoting symmetry or reflection</p> <p>For correct plane</p> <p>Allow “in <math>y</math> coordinates” or “in <math>y</math> axis”</p> <p><b>SR</b> For symmetry implied by reference to opposite signs in <math>y</math> coordinates of <math>C</math> and <math>D</math>, award B1 only</p>
<p>(iii) <math>\cos \theta = \frac{\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right] \cdot \left[1, -\sqrt{3}, \frac{1}{2}\sqrt{2}\right]}{\sqrt{1+3+\frac{1}{2}}\sqrt{1+3+\frac{1}{2}}}</math>  <math>= \frac{\left 1-3+\frac{1}{2}\right }{\frac{9}{2}} = \frac{\frac{3}{2}}{\frac{9}{2}} = \frac{1}{3}</math></p>	<p>M1 A1 M1 A1</p>	<p>For using scalar product of normal vectors</p> <p>For correct scalar product</p> <p>For product of both moduli in denominator</p> <p>For correct answer. Allow <math>-\frac{1}{3}</math></p>
<b>11</b>		
<p>6 (i) <math>(m^2 + 16 = 0 \Rightarrow) m = \pm 4i</math></p> $\text{CF} = A \cos 4x + B \sin 4x$	<p>M1 A1</p>	<p>For attempt to solve correct auxiliary equation (may be implied by correct CF)</p> <p>For correct CF</p> <p>(<b>AETrig</b> but not <math>Ae^{4ix} + Be^{-4ix}</math> only)</p>
<p>(ii) <math>\frac{dy}{dx} = p \sin 4x + 4px \cos 4x</math></p> $\frac{d^2y}{dx^2} = 8p \cos 4x - 16px \sin 4x$ $\Rightarrow 8p \cos 4x = 8 \cos 4x$ $\Rightarrow p = 1$ $\Rightarrow (y =) A \cos 4x + B \sin 4x + x \sin 4x$	<p>M1 A1 A1√ M1 A1 B1√</p>	<p>For differentiating PI twice, using product rule</p> <p>For correct <math>\frac{dy}{dx}</math></p> <p>For unsimplified <math>\frac{d^2y}{dx^2}</math>. f.t. from <math>\frac{dy}{dx}</math></p> <p>For substituting into DE</p> <p>For correct <math>p</math></p> <p>For using <math>\text{GS} = \text{CF} + \text{PI}</math>, with 2 arbitrary constants in CF and none in PI</p>

<b>(iii)</b>	$(0, 2) \Rightarrow A = 2$	B1√	For correct $A$ . f.t. from their GS
	$\frac{dy}{dx} = -4A \sin 4x + 4B \cos 4x + \sin 4x + 4x \cos 4x$	M1	For differentiating their GS
	$x = 0, \frac{dy}{dx} = 0 \Rightarrow B = 0$	M1	For substituting values for $x$ and $\frac{dy}{dx}$
	$\Rightarrow y = 2 \cos 4x + x \sin 4x$	A1 4	to find $B$ For stating correct solution <b>CAO</b> including $y =$
<b>12</b>			
<b>7 (i)</b>	$\cos 6\theta = 0 \Rightarrow 6\theta = k \times \frac{1}{2}\pi$	M1	For multiples of $\frac{1}{2}\pi$ seen or implied
	$\Rightarrow \theta = \frac{1}{12}\pi \{1, 3, 5, 7, 9, 11\}$	A1	A1 for any 3 correct
		A1 3	A1 for the rest, and no extras in $0 < \theta < \pi$
<b>(ii)</b>	<b>METHOD 1</b>		
	$\text{Re}(c+is)^6 = \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	M1	For expanding $(c+is)^6$ at least 4 terms and 2 binomial coefficients needed
	$\cos 6\theta = c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$	A1	For 4 correct terms
	$\Rightarrow \cos 6\theta = 32c^6 - 48c^4 + 18c^2 - 1$	M1	For using $s^2 = 1 - c^2$
	$\Rightarrow \cos 6\theta = (2c^2 - 1)(16c^4 - 16c^2 + 1)$	A1	For correct expression for $\cos 6\theta$
		A1 5	For correct result <b>AG</b> (may be written down from correct $\cos 6\theta$ )
	<b>METHOD 2</b>		
	$\text{Re}(c+is)^3 = \cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta$	M1	For expanding $(c+is)^3$ at least 2 terms and 1 binomial coefficient needed
	$\Rightarrow \cos 6\theta = \cos 2\theta (\cos^2 2\theta - 3\sin^2 2\theta)$	A1	For 2 correct terms
	$\Rightarrow \cos 6\theta = (2\cos^2 \theta - 1) \left( 4(2\cos^2 \theta - 1)^2 - 3 \right)$	M1	For replacing $\theta$ by $2\theta$
	$\Rightarrow \cos 6\theta = (2c^2 - 1)(16c^4 - 16c^2 + 1)$	A1	For correct expression in $\cos \theta$ (unsimplified)
		A1	For correct result <b>AG</b>
<b>(iii)</b>	<b>METHOD 1</b>		
	$\cos 6\theta = 0$	M1	For putting $\cos 6\theta = 0$
	$\Rightarrow 6$ roots of $\cos 6\theta = 0$ satisfy	A1	For association of roots with quartic and quadratic
	$16c^4 - 16c^2 + 1 = 0$ and $2c^2 - 1 = 0$	B1	For correct association of roots with quadratic
	But $\theta = \frac{1}{4}\pi, \frac{3}{4}\pi$ satisfy $2c^2 - 1 = 0$	M1	For using product of 4 roots <b>OR</b> for solving quartic
	<i>EITHER</i> Product of 4 roots <i>OR</i> $c = \pm \frac{1}{2}\sqrt{2 \pm \sqrt{3}}$	A1 5	For correct value (may follow A0 and B0)
	$\Rightarrow \cos \frac{1}{12}\pi \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cos \frac{11}{12}\pi = \frac{1}{16}$		

METHOD 2

$\cos 6\theta = 0$	M1	For putting $\cos 6\theta = 0$
$\Rightarrow 6$ roots of $\cos 6\theta = 0$ satisfy	A1	For association of roots with sextic
$32c^6 - 48c^4 + 18c^2 - 1 = 0$		
Product of 6 roots $\Rightarrow$	M1	For using product of 6 roots
$\cos \frac{1}{12}\pi \cdot \frac{1}{\sqrt{2}} \cdot \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cdot \frac{-1}{\sqrt{2}} \cdot \cos \frac{11}{12}\pi = -\frac{1}{32}$	B1	For using $\cos\left\{\frac{3}{12}\pi, \frac{9}{12}\pi\right\} = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$
$\cos \frac{1}{12}\pi \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cos \frac{11}{12}\pi = \frac{1}{16}$	A1	For correct value

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<b>8 (i)</b>	$g(x) = \frac{1}{2-2 \cdot \frac{1}{2-2x}} = \frac{2-2x}{2-4x} = \frac{1-x}{1-2x}$	M1	For use of $f f(x)$
		A1	For correct expression <b>AG</b>
	$gg(x) = \frac{1 - \frac{1-x}{1-2x}}{1 - 2 \cdot \frac{1-x}{1-2x}} = \frac{-x}{-1} = x$	M1	For use of $gg(x)$
		A1 <b>4</b>	For correct expression <b>AG</b>

<b>(ii)</b>	Order of $f = 4$	B1	For correct order
	order of $g = 2$	B1 <b>2</b>	For correct order

**(iii)** METHOD 1

$y = \frac{1}{2-2x} \Rightarrow x = \frac{2y-1}{2y}$	M1	For attempt to find inverse
$\Rightarrow f^{-1}(x) = h(x) = \frac{2x-1}{2x}$ OR $1 - \frac{1}{2x}$	A1 <b>2</b>	For correct expression

METHOD 2

$f^{-1} = f^3 = f g$ or $g f$	M1	For use of $f g(x)$ or $g f(x)$
$f g(x) = h(x) = \frac{1}{2-2\left(\frac{1-x}{1-2x}\right)} = \frac{1-2x}{-2x}$	A1	For correct expression

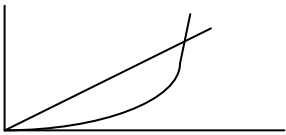
**(iv)**

$\begin{array}{c cccc} & e & f & g & h \\ \hline e & e & f & g & h \\ f & f & g & h & e \\ g & g & h & e & f \\ h & h & e & f & g \end{array}$	M1	For correct row 1 and column 1
	A1	For e, f, g, h in a latin square
	A1	For correct diagonal e - g - e - g
	A1 <b>4</b>	For correct table

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## 4728 Mechanics 1

<b>1 i</b>	$v = 4.2 + 9.8 \times 1.5$ $v = 18.9 \text{ ms}^{-1}$ .	M1 A1 [2]	Uses $v = u + gt$ 18.9(15) from $g = 9.81$
<b>ii</b>	$s = 4.2 \times 1.5 + 9.8 \times 1.5^2/2$ or $18.9^2 = 4.2^2 + 2 \times 9.8s$ $s = 17.325 \text{ m}$	M1 A1 [2]	Uses $s = ut + gt^2/2$ or $v^2 = u^2 + 2gs$ Accept 17.3
<b>iii</b>	$v^2 = 4.2^2 + 2 \times 9.8 \times (17.3(25) - 5)$ $v = 16.1 \text{ ms}^{-1}$	M1 A1 [2]	$18.9^2 = u^2 + 2 \times 9.8 \times 5$ $u = 16.1 \text{ ms}^{-1}$ . Accept answers close to 16.1 from correct working
<b>2 i</b>	Resolves a force in 2 perpendicular directions Uses Pythagoras $R^2 = (12+19\cos60)^2 + (19\sin60)^2$ $R = 27.1 \text{ N}$ { $R = \sqrt{(19+12\cos60)^2 + (12\sin60)^2} = 27.1$ }	M1 DM1 A1 A1 A1 [5]	<i>Diagram for vector addition/subtraction</i>  <i>Uses Cosine Rule</i> $R^2 = 12^2 + 19^2 - 2 \times 12 \times 19\cos120$ $R = 27.1$
<b>ii</b>	Trig on a valid triangle for correct angle $\tan\theta = (19\sin60)/(12 + 19\cos60)$ etc Angle is $37.4^\circ, 37.5^\circ$	M1 A1 A1 [3]	Either Pythagoras or vector add/sub triangle $\sin\theta/19 = \sin120/(27.1)$ etc
<b>3ia</b>	$+/- (9m + 2 \times 0.8)$ { $+/- (3.5 \times 0.8 - 2 \times 0.8)$ } $+/- (-3.5m + 3.5 \times 0.8)$ { $+/- (9m + 3.5m)$ } $+/- (9m + 2 \times 0.8) = +/- (-3.5m + 3.5 \times 0.8)$ $m = 0.096 \text{ kg}$	B1 B1 M1 A1 [4]	Before mom, or mom change Q, OK with g After mom, or mom change P, OK with g Equates moms, or changes, accept with g Do not award if g used
<b>ib</b>	$+/- 0.096(9 +/- 3.5)$ OR $+/- 0.8(3.5 - 2)$ $+/- 1.2 \text{ kgms}^{-1}$	M1 A1ft [2]	Using before & after speeds of P or Q, no g ft $12.5 \times cv(0.096)$
<b>ii</b>	$(0.8+0.4)v$ or $0.8v + 0.4v$ $3.5 \times 0.8 + 0.4 \times 2.75 = (0.8+0.4)v$ $v = 3.25 \text{ ms}^{-1}$	M1 A1 A1 [3]	Using Q and R common speed after, no g $2.8 + 1.1 = 1.2v$
<b>4ia</b>	$0.3g\cos 60$ and $0.3g\sin60$ $0.4g\cos60$ and $0.4g\sin60$ Calculates either relevant difference Perp = $0.1g\cos60$ and Para = $+/- 0.1g\sin60$	B1 B1 M1 A1 [4]	Accept use of "m = 0.1 kg" for M1 and $0.1g\cos60$ (B1) $0.1g\sin60$ (B1) $= 0.49$ and $= 0.849$ (accept 0.85 and 0.84)
<b>ib</b>	$0.1g\sin60 = \mu 0.1g\cos60$ $= 1.73 (= \sqrt{3})$	M1 A1 [2]	$F = \mu R, F > R > 0$ From correct R, F values

4 ii	$0.5g - T = 0.5a$ $T - 0.4g = 0.4a$ $a = 1.09 \text{ ms}^{-2}$ $T = 4.36 \text{ N}$	M1  A1 B1 B1 [4]	N2L for either particle no resolving, at least 1 unknown Formula round the pulley, M0A0. But award M1 for $T - 0.4g = 0.4 \times 1.09$ etc later Both equations correct
5 i	$11 = 3 + 20a$ (a = 0.4) $8 = 3 + (11-3)t/20$ $t = 12.5$	M1 M1 A1 [3]	Uses $v = u + at$ , no zero terms Their $a > 0$ . $t/20 = (8-3)/(11-3)$ is M1M1
ii	$s(A,20) = 8 \times 20 (=160)$ $s(B,20) = (3 + 11) \times 20/2 =$ $3 \times 20 + 0.4 \times 20^2/2 (=140)$ $8T = (3+11) \times 20/2 + 11 \times (T-20)$ or $(160 - 140) = 11t - 8t$ $T = 26 \frac{2}{3}$	B1  B1 M1 A1 A1 [5]	Or $s(A) = 8T$  or as stage of $s(B) = (3+11) \times 20/2 + 11 \times (T-20)$ 3 part equation balancing distances  Accept 26.6 or 26.7
iii		B1  B1  B1 [3]	Linear rising graph (for A) starting at B's start Non-linear rising graph for B below A's initially. Accept 2 straight lines as non-linear. Single valued graphs graphs intersect and continue
6 i	$a = 2 \times 0.006t - 0.18$ $a = 0.012t - 0.18$	M1 A1 [2]	Differentiates $v$ (not $v/t$ ) Award for unsimplified form, accept $+c$ , not $+k$
ii	$0.012t - 0.18 = 0$ $t = 15$  $0.006 \times 15^2 - 0.18 \times 15 + k = 0.65$ $k = 2$	M1* A1 D*M1 A1 A1 AG [5]	Sets $a = 0$ , and solves for $t$  Substitutes $t(v(\min))$ in $v(t)$
iii	$s = 0.006t^3/3 - 0.18t^2/2 + 2t (+c)$ $(s = 0.002t^3 - 0.09t^2 + 2t (+c))$ $t = 0, s = 0$ hence $c = 0$ $L = 0.002 \times 28.4^3 - 0.09 \times 28.4^2 + 2 \times 28.4$ $L = 30.0 \text{ m}$	M1A1  B1 M1 A1 [5]	Integrates $v$ (not multiplies by $t$ ). Award if $+c$ omitted, accept $kt$ Explicit, not implied (or uses limits 0, 28.4) Substitutes 28.4 or 14.2 in $s(t)$ , (and $k=2$ ) Accept a r t 30(.0), accept $+c$



7 i	$(Fr =) 0.15 \times 600g\cos 10$ $(Wt \text{ cmpt} =) 600g\sin 10$ $600 \times 0.11 = T - 0.15 \times 600g\cos 10 - 600g\sin 10$ $(66 = T - 868.6 - 1021)$ $T = 1960 \text{ N}$	B1 B1 M1 A1 A1 [5]	Implied by $Fr = 0.15 \times 600g\cos 10 (=868.6..)$ N2L. T with at least 1 resolved forces and $600 \times 0.11$ 1955.6..
ii a	$a(\text{up}) = +/- (600g\sin 10 + 0.15 \times 600g\cos 10) / 600$ $a(\text{up}) = +/- 3.15 \text{ ms}^{-2}$ AG	M1 A1 [2]	2 resolved forces and 600a or “unit mass” Disregard sign, accept 3.149
b	UP $v^2 = 2 \times 0.11 \times 10$ $v = 1.48$ when cable breaks $t = 1.48 / 3.149$ $(t = 0.471 \text{ time for log to come to rest})$ $s = 1.48^2 / (2 \times 3.149)$ $s = 0.349$ distance for log to come to rest DOWN $a(\text{down}) = (600g\sin 10 - 0.15 \times 600g\cos 10) / 600$ $10 + 0.349 = 0.254t^2 / 2$ $t = 9.025$ $T = (9.025 + 0.471) = 9.5 \text{ s}$	M1 A1 M1 M1 A1 B1 M1 A1 A1 [9]	Correct, need not be accurate Or $1.48 = 0 + 3.15t$ Correct, need not be accurate = 0.254 Needs $a < 3.15, s > 10$ . Or $V^2 = 2 \times 0.254 \times (10 + 0.349)$ [ $V = 2.29..$ ], $V = 0.254t$ Correct, need not be accurate Accept 9.49

## 4729 Mechanics 2

<b>1</b>	$75 \times 9.8 \times 40$ $(75 \times 9.8 \times 40) \div 120$ 245 W	B1 M1 A1 [3]	Average Speed = $40 \div 120$ $(75 \times 9.8) \times (\text{Average speed})$	<b>3</b>
<b>2 (i)</b>	$v^2 = 2 \times 9.8 \times 3$ or $2 \times 9.8 \times 1.8$ $v_1 = \sqrt{6g}$ or $\sqrt{58.8}$ or $\frac{7}{5}\sqrt{30}$ or 7.67 $v_2 = \sqrt{3.6g}$ or $\sqrt{35.28}$ or $\frac{21}{5}\sqrt{2}$ or 5.94 $I = \pm 0.2(5.94 + 7.67)$ 2.72	M1 A1 A1 M1 A1ft [5]	Kinematics or energy Speed of impact ( $\pm$ ) Speed of rebound ( $\pm$ ) +ve, ft on $v_1$ and $v_2$	
<b>(ii)</b>	$e = 5.94/7.67$ $0.775$ or $\frac{\sqrt{15}}{5}$	M1 A1ft [2]	Allow 0.774, ft on $v_1$ and $v_2$	<b>7</b>
<b>3 (i)</b>	$\bar{u} = 0.2$ (from vertex) or 0.8 or 0.1 $0.5\bar{d} = 0.2 \times \bar{u} + 0.3 \times 0.65$ $\bar{d} = 0.47$	B1 M1 A1 A1 [4]	com of conical shell <b>AG</b>	
<b>(ii)</b>	$s = 0.5$ $T \sin 80^\circ \times 0.5 = 0.47 \times 0.5 \times 9.8$ $T = 4.68 \text{ N}$	B1 M1 A1 A1 [4]	slant height, may be implied	<b>8</b>
<b>4 (i)</b>	$D - 400 = 700 \times 0.5$ $D = 750 \text{ N}$	M1 A1 [2]	3 terms	
<b>(ii)</b>	$P = 750 \times 12$ 9 000 W or 9 kW	M1 A1ft [2]		
<b>(iii)</b>	$P/35 = 400$ 14 000 W or 14 kW	M1 A1 [2]		
<b>(iv)</b>	$D = 14000/12$ $3500/3 = 400 + 700 \times 9.8 \sin \theta$ $\theta = 6.42^\circ$	B1ft M1 A1 A1 [4]	May be implied 3 terms Their P/12	<b>10</b>



(iii)	$\varphi = 26.6^\circ$ or $\sin \varphi = \frac{1}{\sqrt{5}}$ or $\cos \varphi = \frac{2}{\sqrt{5}}$ or $\tan \varphi = 0.5$ $T = 0.98$ or $0.1g$ $N \cos \theta = T \sin \varphi + 0.2 \times 9.8$ $N \times 3/5 = 0.438 + 1.96$ $N = 4.00$ $N \sin \theta + T \cos \varphi = 0.2 \times 4 \times \omega^2$ $4 \times 4/5 + 0.98 \cos 26.6^\circ = 0.8 \omega^2$ $\omega = 2.26 \text{ rad s}^{-1}$	B1  B1 M1 A1 A1 M1 A1 A1	$\varphi = \text{angle to horizontal}$  Vertically, 3 terms may be implied Horizontally, 3 terms
		[8]	15

## 4730 Mechanics 3

1	$0.4(3\cos 60^\circ - 4) = -I \cos \theta \quad (= -1)$ $0.4(3\sin 60^\circ) = I \sin \theta \quad (= 1.03920)$ $[\tan \theta = -1.5\sqrt{3}/(1.5 - 4);$ $I^2 = 0.4^2[(1.5 - 4)^2 + (1.5\sqrt{3})^2]$ $\theta = 46.1 \text{ or } I = 1.44$ $I = 1.44 \text{ or } \theta = 46.1$	M1 A1 A1  M1 A1 M1  A1ft [7]	For using $I = \Delta mv$ in one direction  SR: Allow B1 (max 1/3) for $3\cos 60^\circ - 4 = -I \cos \theta$ and $3\sin 60^\circ = I \sin \theta$  For eliminating $I$ or $\theta$ (allow following SR case) Allow for $\theta$ (only) following SR case. For substituting for $\theta$ or for $I$ (allow following SR case) ft incorrect $\theta$ or $I$ ; allow for $\theta$ (only) following SR case.
	Alternatively $I^2 = 1.2^2 + 1.6^2 - 2 \times 1.2 \times 1.6 \cos 60^\circ \quad \text{or}$ $'V'^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \cos 60^\circ$ $I = 1.44$ $\frac{\sin \theta}{3(\text{or } 1.2)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \quad \text{or}$ $\frac{\sin \alpha}{4(\text{or } 1.6)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \text{ and } \theta = 120 - \alpha$ $\theta = 46.1$	M1 A1 M1 A1 M1  A1ft  A1 [7]	For use of cosine rule  For correct use of factor 0.4 (= m)  For use of sine rule  $\alpha$ must be angle opposite 1.6; ( $\alpha = 73.9$ ) ft value of $I$ or ' $V$ '
2	$2a + 3b = 2 \times 4$ $b - a = 0.6 \times 4$ $[2(b - 2.4) + 3b = 8]$ $b = 2.56$ $v = 2.56$	M1 A1 M1 A1 M1 A1 B1ft [7]	For using the principle of conservation of momentum  For using NEL  For eliminating $a$  ft $v = b$
3(i)	$2W(a \cos 45^\circ) = T(2a)$ $W = \sqrt{2} T$	M1 A1 A1 [3]	For using 'mmt of $2W = \text{mmt of } T$ '  AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2} \text{ and } V = T/\sqrt{2} - 2W$ $W(a \cos \alpha) + H(2a \sin \alpha) = V(2a \cos \alpha)$ $[W \cos \alpha - T \sqrt{2} \sin \alpha = T \sqrt{2} \cos \alpha - 4W \cos \alpha]$ $T \sqrt{2} \sin \alpha = (5W - T \sqrt{2}) \cos \alpha$ $\tan \alpha = 4$	B1 M1 A1  M1 A1ft A1 [6]	For taking moments about C for BC  For substituting for H and V and reducing equation to the form $X \sin \alpha = Y \cos \alpha$

	Alternatively for part (ii) anticlockwise mmt = $W(a \cos\alpha) + 2W(2a \cos\alpha + a \cos 45^\circ)$ $= T[2a \cos(\alpha - 45^\circ) + 2a]$ $[5W \cos\alpha + \sqrt{2} W =$ $T(\sqrt{2} \cos\alpha + \sqrt{2} \sin\alpha) + 2]$ $T \sqrt{2} \sin\alpha = (5W - T \sqrt{2}) \cos\alpha$ $\tan\alpha = 4$	M1 A1 A1 M1 A1ft A1 [6]	For taking moments about C for the whole  For reducing equation to the form $X \sin\alpha = Y \cos\alpha$
4(i)	$[-0.2(v + v^2) = 0.2a]$ $[v \, dv/dx = -(v + v^2)]$ $[1/(1 + v)] \, dv/dx = -1$	M1 M1 A1 [3]	For using Newton's second law For using $a = v \, dv/dx$ AG
(ii)	$\ln(1 + v) = -x + C$ $\ln(1 + v) = -x + \ln 3$ $[(1 + dx/dt)/3 = e^{-x} \rightarrow dx/dt = 3e^{-x} - 1$ $\rightarrow e^x \, dx/dt = 3 - e^x]$ $[-e^x/(3 - e^x)] \, dx/dt = -1$	M1 A1 A1 M1 A1 [5]	For integrating  For transposing for v and using $v = dx/dt$ AG
(iii)	$[\ln(3 - e^x) = -t + \ln 2]$ $\ln(3 - e^x) = -t + \ln 2$ Value of t is 1.96 (or $\ln\{2 \div (3 - e)\}$ )	M1 A1 A1 [3]	For integrating and using $x(0) = 0$
5(i)	Loss of EE = $120(0.5^2 - 0.3^2)/(2 \times 1.6)$ and gain in PE = $1.5 \times 4$  $v = 0$ at B and loss of EE = gain in PE (= 6) $\rightarrow$ distance AB is 4m	M1 A1 M1 A1 [4]	For using $EE = \lambda x^2/2L$ and $PE = Wh$  For comparing EE loss and PE gain  AG
(ii)	$[120e/1.6 = 1.5]$ $e = 0.02$ Loss of EE = $120(0.5^2 - 0.02^2)/(2 \times 1.6)$ (or $120(0.3^2 - 0.02^2)/(2 \times 1.6)$ ) Gain in PE = $1.5(2.1 - 1.6 - 0.02)$ (or $1.5(1.9 + 1.6 + 0.02)$ loss) $[KE \text{ at max speed} = 9.36 - 0.72$ (or $3.36 + 5.28)]$ $\frac{1}{2} (1.5/9.8)v^2 = 9.36 - 0.72$ Maximum speed is $10.6 \text{ ms}^{-1}$	M1 A1 B1ft B1ft M1 A1 A1 [7]	For using $T = mg$ and $T = \lambda x/L$  ft incorrect e only  ft incorrect e only For using KE at max speed = Loss of EE - Gain (or + loss) in PE
	First alternative for (ii) x is distance AP $[\frac{1}{2} (1.5/9.8)v^2 + 1.5x + 120(0.5 - x)^2/3.2 =$ $120 \times 0.5^2/3.2]$  KE and PE terms correct EE terms correct $v^2 = 470.4x - 490x^2$ $[470.4 - 980x = 0]$ $x = 0.48$ Maximum speed is $10.6 \text{ ms}^{-1}$	M1 A1 A1 A1 M1 A1 A1	For using energy at P = energy at A    For attempting to solve $dv^2/dx = 0$

	<p>Second alternative for (ii)  <math>[120e/1.6 = 1.5]</math>  <math>e = 0.02</math>  <math>[1.5 - 120(0.02 + x)/1.6 = 1.5 \ddot{x}/g]</math></p> <p><math>n = \sqrt{490}</math>  <math>a = 0.48</math>  Maximum speed is <math>10.6 \text{ ms}^{-1}</math></p>	<p>M1  A1  M1    M1  A1    A1  A1</p>	<p>For using <math>T = mg</math> and <math>T = \lambda x/L</math></p> <p>For using Newton's second law  For obtaining the equation in the form  <math>\ddot{x} = -n^2x</math>, using <math>(AB - L - e_{\text{equil}})</math> for  amplitude and using <math>v_{\text{max}} = na</math>.</p>
6(i)	<p>PE gain by P = <math>0.4g \times 0.8 \sin \theta</math>  PE loss by Q = <math>0.58g \times 0.8 \theta</math></p> <p><math>\frac{1}{2} (0.4 + 0.58)v^2 = g \times 0.8(0.58 \theta - 0.4 \sin \theta)</math>  <math>v^2 = 9.28 \theta - 6.4 \sin \theta</math></p>	<p>B1  B1  M1  A1ft  A1  [5]</p>	<p>For using KE gain = PE loss</p> <p>AEF</p>
(ii)	<p><math>0.4g \sin \theta - R = 0.4v^2/0.8</math>  <math>[0.4g \sin \theta - R = 4.64 \theta - 3.2 \sin \theta]</math>  <math>R = 7.12 \sin \theta - 4.64 \theta</math></p>	<p>M1  A1  M1  A1  [4]</p>	<p>For applying Newton's second law to P and  using <math>a = v^2/r</math></p> <p>For substituting for <math>v^2</math>  AG</p>
(iii)	<p><math>R(1.53) = 0.01(48\dots)</math>, <math>R(1.54) = -0.02(9\dots)</math> or  simply <math>R(1.53) &gt; 0</math> and <math>R(1.54) &lt; 0</math></p> <p><math>R(1.53) \times R(1.54) &lt; 0 \Rightarrow 1.53 &lt; \alpha &lt; 1.54</math></p>	<p>M1  A1    M1  A1  [4]</p>	<p>For substituting 1.53 and 1.54 into <math>R(\theta)</math></p> <p>For using the idea that if <math>R(1.53)</math> and  <math>R(1.54)</math> are of opposite signs then R is zero  (and thus P leaves the surface) for some  value of <math>\theta</math> between 1.53 and 1.54.  AG</p>
7(i)	<p><math>T_{AP} = 19.6e/1.6</math> and <math>T_{BP} = 19.6(1.6-e)/1.6</math></p> <p><math>0.5g \sin 30^\circ + 12.25(1.6 - e) = 12.25e</math>  Distance AP is 2.5m</p>	<p>M1  A1  M1  A1ft  A1  [5]</p>	<p>For using <math>T = \lambda e/L</math></p> <p>For resolving forces parallel to the plane</p>
(ii)	<p>Extensions of AP and BP are <math>0.9 + x</math> and  <math>0.7 - x</math> respectively</p> <p><math>0.5g \sin 30^\circ + 19.6(0.7 - x)/1.6</math>  <math>- 19.6(0.9 + x)/1.6 = 0.5 \ddot{x}</math>  <math>\ddot{x} = -49x</math></p> <p>Period is 0.898 s</p>	<p>B1    B1ft  B1  M1  A1  [5]</p>	<p>AG</p> <p>For stating <math>k &lt; 0</math> and using <math>T = 2\pi/\sqrt{-k}</math></p>
(iii)	<p><math>2.8^2 = 49(0.5^2 - x^2)</math>  <math>x^2 = 0.09</math></p> <p><math>x = 0.3</math> and <math>-0.3</math></p>	<p>M1  A1ft  A1    A1ft  [4]</p>	<p>For using <math>v^2 = \omega^2(A^2 - x^2)</math> where <math>\omega^2 = -k</math>  ft incorrect value of k  May be implied by a value of x  ft incorrect value of k or incorrect value of  <math>x^2</math> (stated)</p>

# 4732 Probability & Statistics 1

Note: “(3 sfs)” means “answer which rounds to ... to 3 sfs”. If correct ans seen to  $\geq 3$ sfs, ISW for later rounding  
Penalise over-rounding only once in paper.

<b>1 (i)</b>	attempts at threading indep prob of succeeding in threading const	B1 B1 2	in context in context
<b>(ii) (a)</b>	$0.7^4 \times 0.3$ $= 0.0720$ (3sf)	M1 A1 2	Condone 0.072
<b>(b)</b>	$0.7^5$  $= 0.168$ (3 sfs)	M2  A1 3	or $1 - (0.3 + 0.7 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3 + 0.7^4 \times 0.3)$ M1 for one term omitted or extra or wrong or $1 - 0.7^5$ or $(0.3 + \dots + 0.7^4 \times 0.3)$ or 0.3, 0.7 muddle or $0.7^4$ or $0.7^6$ alone. 0.6 not 0.7 M0 in (a) M1 in (b) 1/3, 2/3 used M1 in (a) M1 in (b)
<b>(iii)</b>	likely to improve with practice  hence independence unlikely or prob will increase each time	B1  B1 2	or thread strands gradually separate 1 <sup>st</sup> B1 must be in context. hence independence unlikely or prob will decrease each time or similar Allow ‘change’
<b>Total</b>		<b>[9]</b>	
<b>2 (i) (a)</b>	Use of correct midpts $\Sigma lf \div \Sigma f$ (= 706 $\div$ 40) $= 17.65$  $\Sigma l^2 f$ (= 13050.5) $\sqrt{\frac{13050.5}{40} - 17.65^2}$ (= $\sqrt{14.74}$ ) $= 3.84$ (3 sfs)	B1 M1 A1  M1 M1 A1 6	11,14,18,25.5 $l$ within class, $\geq$ three $lf$ seen [17.575,17.7]  $\geq$ three $l^2 f$ seen  $\div 40, -\text{mean}^2, \sqrt{\text{Dep}} > 0.$ $\Sigma (l - 17.65)^2 f$ , at least 3 M1, $\div 40, \sqrt{\quad}$ M1, 3.84 A1. $\div 4 \Rightarrow \text{max B1M0A0M1M0A0}$
<b>(b)</b>	mid pts used or data grouped or exact values unknown oe	B1 1	not “orig values were guesses”
<b>(ii)</b>	$20 \div 5$ $= 4$	M1 A1 2	condone $20 \div [4,5]$ or ans 5
<b>(iii)</b>	$20.5^{\text{th}}$ value requ’d and 1 <sup>st</sup> two classes contain 14 values $16 - 20$	M1 B1 2	condone $20^{\text{th}}$ oe or third class oe
<b>(iv) (a)</b>	increase	B1 1	
<b>(b)</b>	decrease	B1 1	
<b>Total</b>		<b>[13]</b>	
<b>3 (i)</b>	$S_{hm} = 0.2412$ $S_{hh} = 0.10992$ $S_{mm} = 27.212$ $r = \frac{S_{hm}}{\sqrt{(S_{hh} S_{mm})}}$ $= 0.139$ (3 sfs)	B1 M1  A1 3	Allow x or $\div 5$  any one $S$ correct fit their $S$ s
<b>(ii)</b>	Small, low or not close to 1 or close to 0 oe pts not close to line oe	B1 ft B1	1 <sup>st</sup> B1 about value of $r$ 2 <sup>nd</sup> B1 about diag
<b>(iii)</b>	none or unchanged or “0.139” oe	B1 1	
<b>(iv)</b>	Larger oe	B1 1	
<b>Total</b>		<b>[7]</b>	



<b>4 (i)</b>	$(0 \times \frac{1}{2}) + 1 \times \frac{1}{4} + 2 \times \frac{1}{8} + 3 \times \frac{1}{8}$ $= \frac{7}{8}$ or 0.875 oe $(0 \times \frac{1}{2}) + 1 \times \frac{1}{4} + 2^2 \times \frac{1}{8} + 3^2 \times \frac{1}{8}$ (= $1 \frac{7}{8}$ ) $- (\frac{7}{8})^2$ $= \frac{71}{64}$ or 1.11 (3 sfs) oe	M1 A1 M1 M1 A1 5	$\geq 2$ non-zero terms seen If $\div 3$ or 4 M0M0M1(poss) $\geq 2$ non-zero terms seen dep +ve result M1 all4 $(x-0.875)^2$ terms seen. M1 mult p, $\Sigma$ A1 1.11
<b>(ii)</b>	Bin stated or implied 0.922 (3 sfs)	M1 A1 2	Eg table or $\frac{1}{4}^n \times \frac{3}{4}^m$ ( $n+m=10, n, m \neq 1$ ) or 10C4 or 5(or 4 or 6) terms correct
<b>(iii)</b>	$n = 10$ & $p = \frac{1}{8}$ stated or implied ${}^{10}C_4 \times \frac{7}{8}^6 \times \frac{1}{8}^4$ $= 0.0230$ (3 sfs)	M1 M1 A1 3	condone 0.023
<b>Total</b>		<b>[10]</b>	
<b>5 (i)</b>	$\frac{6}{14} \times \frac{5}{13} \times \frac{3}{12}$ $\times 3!$ oe $= \frac{45}{182}$ or 0.247 (3 sfs)oe	M1 M1 A1 3	${}^6C_1 \times {}^5C_1 \times {}^3C_1$ $\div {}^{14}C_3$ With repl M0M1A0
<b>(ii)</b>	$\frac{6}{14} \times \frac{5}{13} \times \frac{4}{12} + \frac{5}{14} \times \frac{4}{13} \times \frac{3}{12} + \frac{3}{14} \times \frac{2}{13} \times \frac{1}{12}$ $= \frac{31}{364}$ or 0.0852 (3 sf)	M2 A1 3	${}^6C_3 + {}^5C_3 + {}^3C_3$ M1 for any one $(\div {}^{14}C_3)$ M1 all 9 numerators correct. With repl M1 $(6/14)^3 + (5/14)^3 + (3/14)^3$
<b>Total</b>		<b>[6]</b>	
<b>6 (a)</b>	A: diag or explanation showing pts close to st line, always increasing B: Diag or expl based on $r=1 \Rightarrow$ pts on st line $\Rightarrow r(s)=1$	B1 B1 B1 3	. Diag or expl based on $r(s) \neq 1 \Rightarrow$ pts not on st line $\Rightarrow r \neq 1$ $r=1 \Rightarrow$ pts on st line & $r(s) \neq 1 \Rightarrow$ pts not on st line B1B1 $r=1 \Rightarrow r(s)=1$ B2
<b>(b)</b>	$\bar{y} = 2.4 \times 4.5 + 3.7$ $= 14.5$ $4.5 = 0.4 \times "14.5" - c$ $c = 1.3$  $a^2 = x - b^2 y \therefore -14.5$ M1A1; then $a^2 = 4.5 - 0.4 \times 14.5 = -1.3$ M1A1	M1 A1 M1 A1 4	Attempt to sub expression for y $x = 0.96x + 1.48 - c$ oe sub $x = 4.5$ and solve $c = 1.3$  14.5 M1A1. $(y - 3.7)/2.4 = 0.4y - c$ and sub 14.5 M1 $c = 1.3$ A1
<b>Total</b>		<b>[7]</b>	
<b>7 (i)</b>	$\frac{25}{37}$	B2 2	B1 num, B1 denom 25/37xp B1
<b>(ii)</b>	$\frac{15}{23}$ seen or implied $\times \frac{39}{59}$ seen or implied $= \frac{585}{1357}$ or 0.431 (3 sfs) oe	M1 M2 A1 4	M1 num, M1 denom Allow M1 for $39/59x$ or + wrong p
<b>Total</b>		<b>[6]</b>	

<b>8 (i)</b>	$5!_2$ = 60	M1 A1 2	Allow 5P3
<b>(ii)</b>	4! = 24	M1 A1 2	Allow 2×4!
<b>(iii)</b>	${}^2_5 \times {}^3_4$ or $3/5 \times 2/4$ × 2 = ${}^3_5$ oe	M1 M1 A1 3	allow M1 for ${}^2_5 \times {}^3_5 \times 2$ or ${}^{12}_{25}$ or $(6 \times 3!) \div (\mathbf{i})$ M2 or $3! \div (\mathbf{i}), 6 \div (\mathbf{i}), (6+6) \div (\mathbf{i}), 6k \div (\mathbf{i})$ or $6 \times 6$ or 36 or 1-correct answer M1 (k, integer ≤ 5)
<b>Total</b>		<b>[7]</b>	
<b>9 (i)</b>	$p^2$	B1 1	
<b>(ii)</b>	$(q^2p)^2$ oe = AG	B1 1	
<b>(iii)</b>	$r=q^2$  a/(1-r) used $(S_\infty =) \frac{p^2}{1-q^2}$  $= \frac{p^2}{1-(1-p)^2}$ p/(2-p) AG	B1  M1  A1  M1  A1 5	May be implied  With $a=p^2$ and $r=q^2$ or $q^4$  Attempt to simplify using $p+q=1$ correctly. Dep on $r = q^2$ or $q^4$ $\frac{(1-q)^2}{(1-q)(1+q)}$ or $p^2/p(1+q)$ Correctly obtain given answer showing at least one intermediate step.
<b>P2Total</b>		<b>[7]</b>	

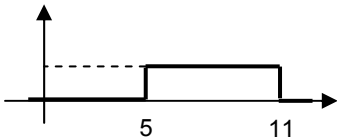
**Total 72 marks**

## 4733 Probability & Statistics 2

Penalise over-specified answers (> 6 SF) first time but only once per paper.

Use **A** or **C** to annotate “over-assertive” or “no context” respectively

<b>1</b>	$\hat{\mu} = \bar{x} = 15.16$ $\hat{\sigma}^2 = \frac{5}{4}s^2$  <b>= 1.363</b>	B1	<b>4</b>	15.16 or 15.2 as answer only	
		M1		Use $\frac{\sum x^2}{5} - \bar{x}^2$ [=1.0904]	
		M1		Multiply by 5/4, or equiv for single formula	
		A1		Final answer 1.36 or 1.363 only, <i>not isw</i>	
<b>2</b>	(i)	Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	<b>2</b>	Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
	(ii)	Ignore random numbers greater than 799, or 399	B1	<b>1</b>	Any valid resolution of this problem, no spurious reasons
<b>3</b>	B(60, 0.35) $\approx$ N(21, 13.65) $\Phi\left(\frac{18.5-21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$ <b>= 1 - 0.7507</b>  <b>= 0.2493</b>	M1	<b>6</b>	B(60, 0.35) stated or implied	
		M1		N(21, ...)	
		A1		Variance or SD = 13.65	
		M1		Standardise, their $np$ and $\sqrt{npq}$ or $npq$ , wrong or no cc	
		A1		Both $\sqrt{npq}$ and cc correct	
A1	Answer, a.r.t. 0.249				
<b>4</b>	$H_0 : \mu = 60; H_1 : \mu < 60$ ( $\alpha$ ) $z = \frac{58.9-60}{\sqrt{5^2/80}} = -1.967$  $< -1.645$	B2	<b>7</b>	Both correct, B2	
		M1		B1 for one error, but not $x$ , $t$ , $\bar{x}$ or $\bar{t}$	
		A1		Standardise 58.9 & $\sqrt{80}$ , allow – or $\sqrt{}$ errors	
		B1		$z$ , art $-1.97$ or $p$ in range [0.024, 0.025] Explicit comparison with $-1.645$ or $0.05$ , or $+1.645$ or $0.95$ if $1.967$ or $0.976$ used	
or:	( $\beta$ ) $c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ <b><math>58.9 &lt; 59.08</math></b>	M1 B1 A1✓		$60 - z \times 5/\sqrt{80}$ , any $z = \Phi^{-1}$ , allow $\sqrt{}$ errors or $\pm$ , not just +; $z = 1.645$ and compare 58.9 59.1 or better, ✓ on wrong $z$	
	Reject $H_0$  Significant evidence that people underestimate time	M1 A1✓		Correct first conclusion, needs essentially correct method including $\sqrt{80}$ or $80$ Contextualised, uncertainty acknowledged SR: $\mu = 58.9$ : B0M1A0B1 max 2/7 SR: 2-tail: max 5/7	
<b>5</b>	(i)	$H_0 : \lambda = 11.0$ $H_1 : \lambda > 11.0$ ( $\alpha$ ) $P(\geq 19) = 1 - 0.9823$ <b>= 0.0177</b> <b>&lt; 0.05</b>	B2 M1 A1 B1	<b>7</b>	Allow $\mu$ . Both correct, B2 One error: B1, but not $C$ , $x$ etc Find $P(\geq 19)$ [or $P(< 19)$ if later 0.95] art 0.0177 [0.9823, ditto] Compare 0.05 [0.95 if consistent], needs M1
		( $\beta$ ) $CR \geq 18$ ,  $P(\geq 18) = 0.0322$ <b><math>19 &gt; 18</math></b>	M1 A1 B1		CR or CV 16/17/18/19 stated or clearly implied, but not < 18 and 0.0322 both seen, allow 0.9678 Explicit comparison with 19, needs M1
		Reject $H_0$  Significant evidence of an increase in number of customers	M1 A1✓		Needs essentially correct method & comparison Contextualised, uncertainty acknowledged SR: Normal, or $P(= 19)$ or $P(\leq 19)$ or $P(> 19)$ : First B2 only.
		(ii)	Can't deduce cause-and-effect, or there may be other factors		B1

6	(i)	(a) Probabilities don't total 1	B1	1	Equivalent statement	
		(b) $P(> 70)$ must be $< P(> 50)$	B1	1	Equivalent statement	
		(c) $P(> 50) = 0.3 \Rightarrow \mu < 50$ $P(< 70) = 0.3 \Rightarrow \mu > 70$	B1	1	Any relevant valid statement, e.g. " $P(< 50) = 0.7$ but $P(< 50)$ must be $< P(< 70)$ "	
	(ii)	$\mu = 60$ by symmetry $\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$ $\sigma = 10/0.5243$ <b>= 19.084</b>	B1 M1  B1 A1	   <b>4</b>	$\mu = 60$ obtained at any point, allow from $\Phi$ One standardisation, equate to $\Phi^{-1}$ , not 0.758 $\Phi^{-1} \in [0.524, 0.5245]$ seen $\sigma$ in range [19.07, 19.1], e.g. 19.073	
7	(i)		M1 A1	 <b>2</b>	Horizontal line Evidence of truncation <i>[no need for labels]</i>	
		(ii)	$\mu = 8$ $\int_5^{11} \frac{1}{6} t^2 dt = \left[ \frac{1}{18} t^3 \right]_5^{11} \quad [= 67]$ $- 8^2$ <b>= 3</b>	B1 M1 B1 M1 A1	   <b>5</b>	8 only, cwd Attempt $\int kt^2 dt$ , limits 5 and 11 seen $k = 1/6$ stated or implied Subtract their (non-zero) mean <sup>2</sup> Answer 3 only, <i>not</i> from MF1
		(iii)	$N(8, 3/48)$ $1 - \Phi\left(\frac{8.3 - 8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$ $= 1 - 0.8848$ <b>= 0.1151</b> Normal distribution only approx.	M1 A1 A1 M1  A1 B1	     <b>6</b>	Normal stated or implied Mean 8 Variance their (non-zero) (ii)/48 Standardise, $\sqrt{n}$ , ignore sign or $\sqrt{\quad}$ errors. cc: M0 Answer, art 0.115 Any equivalent comment, e.g. CLT used
8	(i)	$P(\leq 4) = 0.0473$ Therefore CR is $\leq 4$ $P(\text{Type I error}) =$ <b>4.73%</b>	M1 B1 A1	  <b>3</b>	$P(\leq r)$ from $B(10, 0.7)$ , $r = 3/4/5$ , <i>not</i> N " $\leq 4$ " stated, not just "4", nothing else Answer, art 0.0473 or 4.73%, must be stated	
		(ii)	$B(10, 0.4)$ and find $P(> 4)$ $1 - P(\leq 4)$ <b>= 0.3669</b>	M1 M1 A1	  <b>3</b>	Must be this, <i>not</i> isw, ✓ on (i) Allow for 0.6177 or 0.1622 Answer, art 0.367
		(iii)	$0.5 \times 0.3669$ <b>= 0.18345</b>	M1 A1 ✓	 <b>2</b>	$0.5 \times$ (ii) Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	2	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119												
	(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ [0.7720 - 0.5760] $= \mathbf{0.196}$	M1 M1 A1	3	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196												
	(iii)	Po(60) $\approx$ N(60, 60) $\Phi\left(\frac{69.5-60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	5	N(60, ...) Variance or SD 60 Standardise, $\lambda$ & $\sqrt{\lambda}$ , allow $\lambda$ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890												
	(iv)	(a) $1 - e^{-3m}(1 + 3m)$	M1 A1	2	M1 for one error, e.g. no "1 -", or extra term, or 0 <sup>th</sup> term missing; answer, aesf												
		(b) $m = 1.29,$ $p = 0.89842$ $m = 1.3, p = 0.9008$  Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	4	Substitute 1.29 or 1.3 into appropriate fn <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Comp</th> <th>0.9</th> <th>0.1</th> <th>0</th> </tr> </thead> <tbody> <tr> <td>1.29</td> <td>0.898</td> <td>0.10158</td> <td>-.00158</td> </tr> <tr> <td>1.3</td> <td>0.901</td> <td>0.09918</td> <td>.0008146</td> </tr> </tbody> </table> Explicit comparison with relevant value, & conclusion, needs both $ps$ correct	Comp	0.9	0.1	0	1.29	0.898	0.10158	-.00158	1.3	0.901	0.09918	.0008146
	Comp	0.9	0.1	0													
1.29	0.898	0.10158	-.00158														
1.3	0.901	0.09918	.0008146														
or	Method for iteration; 1.296... 1.2965 or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296... Need at least 4 dp for M1A2													

# 4734 Probability & Statistics 3

<p><b>1(i)</b></p> $\int_{-a}^0 \frac{2}{5} dx + \int_0^{\infty} \frac{2}{5} e^{-2x} dx = 1$ $2a/5 + 1/5 = 1$ $a = 2$ <hr/> <p><b>(ii)</b></p> <p>-</p> $\int_{-2}^0 \frac{2}{5} x dx + \int_0^{\infty} \frac{2}{5} x e^{-2x} dx$ $\int_{-2}^0 \frac{2}{5} x dx = -\frac{a^2}{5}$ $\int_0^{\infty} \frac{2}{5} x e^{-2x} dx = \left[ -\frac{1}{5} x e^{-2x} \right] + \left[ -\frac{1}{10} e^{-2x} \right]$ $= -0.7$	<p>M1</p> <p>A1</p> <p>A1     <b>3</b></p> <hr/> <p>M1</p> <p>A1 ✓</p> <p>M1</p> <p>A1</p> <p>A1     <b>5</b></p> <p style="text-align: right;"><b>[8]</b></p>	<p>Sum of probabilities = 1</p> <hr/> <p><math>\Sigma \int x f(x) dx</math></p> <p>✓ a</p> <p>By parts with 1 part correct Both parts correct CAO</p>
<p><b>2(i)</b></p> <p>4 cartons: Total, <math>Y \sim N(2016, 36)</math>  <math>P(Y \leq 2000) = \Phi(-16/\sqrt{36})</math>  <math>= 0.00383</math></p> <hr/> <p><b>(ii)</b></p> <p><math>E(V) = 0</math></p> <p><math>\text{Var}(V) = 36 + 16 \times 9</math>  <math>= 180</math></p> <hr/> <p><b>(iii)</b></p> <p>0.5</p>	<p>B1B1</p> <p>M1</p> <p>A1     <b>4</b></p> <hr/> <p>B1</p> <p>M1</p> <p>A1     <b>3</b></p> <hr/> <p>B1     <b>1</b></p> <p style="text-align: right;"><b>[8]</b></p>	<p>Mean and variance</p> <hr/> <p>CWO</p>
<p><b>3(i)</b></p> <p>Normal distribution  Mean <math>\mu_1 - \mu_2</math>; variance <math>2.47/n_1 + 4.23/n_2</math></p> <hr/> <p><b>(ii)</b></p> <p><math>H_0: \mu_1 = \mu_2, H_1: \mu_1 \neq \mu_2</math>  <math>(9.65 - 7.23)/\sqrt{(2.47/5 + 4.23/10)}</math>  <math>= 2.527</math></p> <p><math>&gt; 2.326</math>  Reject <math>H_0</math>  There is sufficient evidence at the 2% significance level that the means differ</p> <hr/> <p><b>(iii)</b></p> <p>Any relevant comment.</p>	<p>B1</p> <p>B1B1</p> <p style="text-align: center;"><b>3</b></p> <hr/> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>A1     <b>6</b></p> <hr/> <p>B1     <b>1</b></p> <p style="text-align: right;"><b>[10]</b></p>	<hr/> <p>Or find critical region  Numerator</p> <p>Compare with critical value  <b>SR1: If no specific comparison but CV and conclusion correct B1. Same in Q5,6,7</b>  <b>SR2: From CI: <math>2.42 \pm z\sigma</math> M1, <math>\sigma</math> correct</b>  <b><math>z = 2.326</math> B1, (0.193, 4.647) A1</b>  <b>0 in not in CI; reject <math>H_0</math> etc M1A1 Total 6</b>  <b>Conclusions not over-assertive in Q3, 5, 6</b></p> <hr/> <p>e.g sample sizes too small for CLT to apply</p>

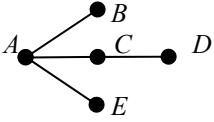
<p><b>4(i)</b></p>	$G(y) = P(Y \leq y) = P(1/(1+V) \leq y)$ $= P(V \geq 1/y - 1)$ $= 1 - F(1/y - 1)$ $= \begin{cases} 0 & y \leq 0, \\ 8y^3 & 0 < y \leq 1/2, \\ 1 & y > 1/2. \end{cases}$ $g(y) = \begin{cases} 24y^2 & 0 < y \leq 1/2, \\ 0 & \text{otherwise.} \end{cases}$ <hr/> $\int 24y^2/y^2 dy \text{ with limits}$ $= 12$	<p>M1 A1 A1</p> <p>A1 B1</p> <p>M1 A1     <b>7</b></p> <hr/> <p>M1 A1     <b>2</b></p> <p><b>[9]</b></p>	<p>Use of F</p> <p><math>8y^3</math> obtained correctly Correct range. Condone omission of <math>y \leq 0</math></p> <p>For <math>G'(y)</math> Correct answer with range <math>\sqrt{\quad}</math></p> <hr/> <p>With attempt at integration</p>
<p><b>5(i)</b></p>	<p>Use <math>p_s \pm z\sqrt{(p_s q_s/200)}</math> <math>z = 1.645</math> <math>s = \sqrt{(0.135 \times 0.865/200)}</math> (0.0952, 0.1747)</p> <hr/> <p><b>(ii)</b> <math>H_0: p_1 - p_2 = 0, H_1: p_1 - p_2 &gt; 0</math> <math>27/200 - 8/100</math> <math>\sqrt{35/300 \times 265/300 \times (200^{-1} + 100^{-1})}</math> <math>= 1.399</math> <math>&gt; 1.282</math> Reject <math>H_0</math>. There is sufficient evidence at the 10% significance level that the proportion of faulty bars has reduced</p>	<p>M1 B1 A1 A1     <b>4</b></p> <hr/> <p>B1 M1 B1 A1 A1 M1</p> <p>A1     <b>7</b></p> <p><b>[11]</b></p>	<p>Or /199 (0.095, 0.175) to 3DP</p> <hr/> <p>Or equivalent Correct form. Pooled estimate of <math>p = 35/300</math> Correct form of sd</p> <p>OR: <math>P(z \geq 1.399) = 0.0809 &lt; 0.10</math> <b>SR: No pooled estimate: B1M1B0B0</b> <b>A1 for 1.514, M1A1 Max 5/7</b></p>
<p><b>6(i)</b></p>	<p>Assumes that decreases have a normal distn <math>H_0: \mu_{O-F} = 0.2</math> (or <math>\geq</math>), <math>H_1: \mu_{O-F} &gt; 0.2</math> O-F: 0.6 0.4 0.2 0.1 0.3 0.2 0.4 0.3 <math>\bar{D} = 0.3125</math>     <math>s^2 = 0.024107</math> <math>(0.3125 - 0.2) / \sqrt{(0.024107/8)}</math> <math>= 2.049</math> <math>&gt; 1.895</math> Reject <math>H_0</math> – there is sufficient evidence at the 5% significance level that the reduction is more than 0.2</p> <hr/> <p><b>(ii)</b> <math>0.3125 \pm t \sqrt{(0.024107/8)}</math> <math>t = 2.365</math> (0.1827, 0.4423)</p>	<p>B1 B1 M1 B1 A1 M1 A1 M1 A1     <b>9</b></p> <hr/> <p>M1 B1 A1     <b>3</b></p> <p><b>[12]</b></p>	<p>B1 Use paired differences <math>t</math>-test</p> <p>Must have /8</p> <p>OR: <math>P(t \geq 2.049) = 0.0398 &lt; 0.05</math> Allow M1 from <math>t_{14} = 1.761</math> <b>SR: 2-sample test: B1B1M0B1A0</b> <b>M1 using 1.761 A0 Max 4/9</b></p> <hr/> <p>Allow with <math>z</math> but with /8</p> <p>Rounding to (0.283, 0.442)</p>

7(i)	<p>H<sub>0</sub>:Vegetable preference is independent of gender  H<sub>1</sub>: All alternatives</p> <p>E-Values    26    16.25    22.75                    22    13.75    19.25</p> $\chi^2 = 5^2(26^{-1} + 22^{-1}) + 7.25^2(16.25^{-1} + 13.75^{-1}) + 2.25^2(22.75^{-1} + 19.25^{-1})$ $= 9.641$ <p>9.64 &gt; 5.991  Reject H<sub>0</sub>, (there is sufficient evidence at the 5% that) vegetable preference and gender are not independent</p>	<p>B1</p> <p>M1  A1  M1  A1  A1</p> <p>M1  A1</p> <p style="text-align: center;"><b>8</b></p>	<p>For both hypotheses</p> <p>At least one correct  All correct  Correct form of any one  All correct  ART 9.64</p> <p>OR: P(≥ 9.641)=0.00806 &lt;0.05</p>
(ii)	<p>-  (H<sub>0</sub>: Vegetables have equal preference  H<sub>1</sub>: All alternatives)</p> <p>Combining rows: 48    30    42  E-Values:            40    40    40</p> $\chi^2 = (8^2 + 10^2 + 2^2)/40$ $= 4.2$ <p>4.2 &lt; 4.605  Do not reject H<sub>0</sub>, there is insufficient evidence at the 10% significance level of a difference in the proportion of preferred vegetables</p>	<p>M1  A1</p> <p>M1  A1</p> <p>M1</p> <p>A1    <b>6</b></p> <p><b>[14]</b></p>	<p>OR: P(≥ 4.2) = 0.122 &gt; 0.10</p> <p>AEF in context</p>



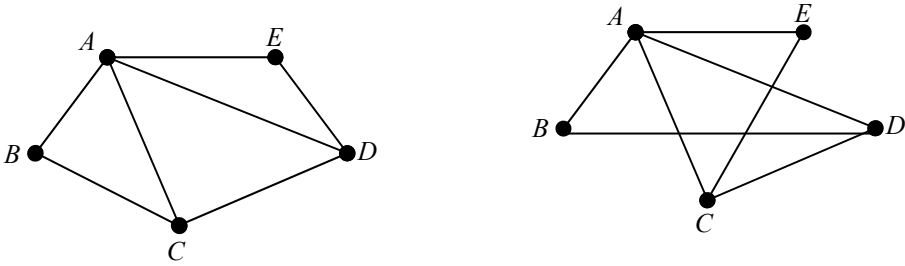
# 4736 Decision Mathematics 1

TO BE ANSWERED ON INSERT				
<b>1</b>	<b>(i)</b>	<p>Path: <math>A - B - C - D - E - F</math> Weight: 9</p>	<p>M1 Evidence of updating at <math>C, D, E</math> or <math>F</math> A1 All temporary labels correct, with no extras</p> <p>B1 All permanent labels correct</p> <p>B1 cao B1 cao</p>	<b>[5]</b>
	<b>(ii)</b>	<p>Total weight of all arcs = 25</p> <p>Only odd nodes are <math>B</math> and <math>E</math>. Least weight path joining <math>B</math> to <math>E</math> is <math>B - C - E = 3</math>.</p> <p>Weight: 28 Route: (example) <math>A - B - D - F - E - C - B - C - D - E - D - C - A</math></p>	<p>B1 Total weight = 25 (may be implied from weight)</p> <p>M1 <math>B</math> to <math>E = 3</math></p> <p>A1 28 (cao)</p> <p>B1 A valid closed route that uses <math>BC, CD</math> and <math>DE</math> twice and all other arcs once</p>	<b>[4]</b>
	<b>(iii)</b>	<p><math>A - B - E - F</math></p> <p>Graph is now Eulerian, so no need to repeat arcs</p>	<p>B1 cao</p> <p>B1 Eulerian (or equivalent)</p>	<b>[2]</b>
<b>Total =</b>			<b>11</b>	

2	(i)	A graph cannot have an odd number of odd vertices (nodes)	B1	Or equivalent (eg $3 \times 5 = 15 \Rightarrow 7\frac{1}{2}$ arcs) Not from a diagram of a specific case	[1]	
	(ii)	It has exactly two odd nodes eg $C A B C D E A D$	B1 B1	2 odd nodes A valid semi-Eulerian trail	[2]	
	(iii)	$AE = 2$ $AC = 3$ $AB = 5$ $CD = 7$  Weight = 17		B1 B1 B1	Correct tree (vertices must be labelled) Order of choosing arcs in a valid application of Prim, starting at A (working shown on a network or matrix) 17	[3]
	(iv)	Lower bound = 29 $A - E - D - F - C - B - A$ = 34 $F - C - A - E - D$ and $F - D - C - A - E$ Vertex B is missed out	B1 M1 A1 B1	29 or 12 + their tree weight from (iii) $A - E - D - F - C -$ 34, from correct working seen Correctly explaining why method fails, need to have explicitly considered both cases	[4]	
<b>Total = 10</b>						

For reference

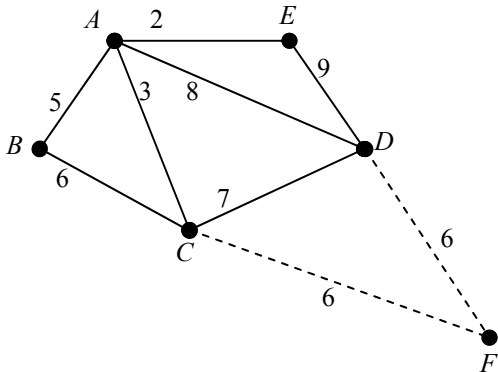
(ii)



(iii) (iv)

	A	B	C	D	E
A	-	5	3	8	2
B	5	-	6	-	-
C	3	6	-	7	-
D	8	-	7	-	9
E	2	-	-	9	-

$CF = 6$   
 $DF = 6$





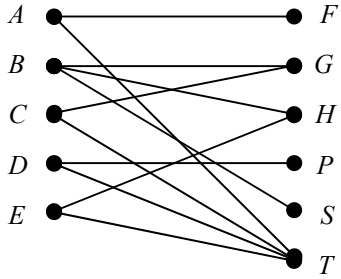
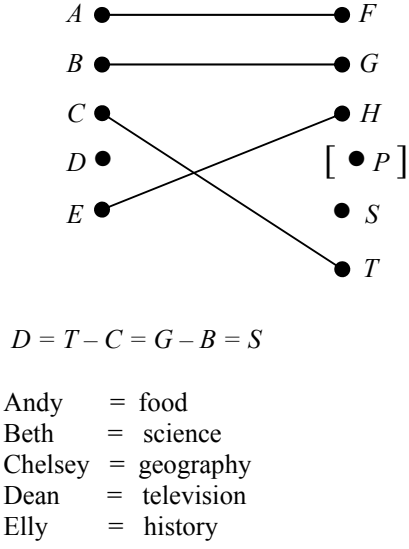
4	(i)	<table border="1"> <tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>D</td><td>D</td><td>D</td><td>D</td><td>C</td></tr> <tr><td>C</td><td>C</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td></tr> </table>	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	D	D	D	D	C	C	C	B	B	B	B	B	B	B	B	B1	15 A's, 4 D's, 3 C's, 8B's (but not just A D C B)	[5]															
		A	A	A	A	A	A	A	A	A	A																																							
		A	A	A	A	A	D	D	D	D	C																																							
C	C	B	B	B	B	B	B	B	B																																									
<table border="1"> <tr><td>Box 1</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>Box 2</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>Box 3</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>Box 4</td><td>D</td><td>D</td><td>D</td><td>D</td><td>C</td><td>C</td><td>C</td><td>B</td></tr> <tr><td></td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td></td></tr> </table>	Box 1	A	A	A	A	A	Box 2	A	A	A	A	A	Box 3	A	A	A	A	A	Box 4	D	D	D	D	C	C	C	B		B	B	B	B	B	B	B		M1	Three boxes each containing A A A A A (or shown using weights)												
Box 1	A	A	A	A	A																																													
Box 2	A	A	A	A	A																																													
Box 3	A	A	A	A	A																																													
Box 4	D	D	D	D	C	C	C	B																																										
	B	B	B	B	B	B	B																																											
<p>Cannot fit all the items into box 4 There is only room for one B in a box</p>	A1	A box containing all the rest Completely correct, including order of packing into boxes																																																
	(ii)	<table border="1"> <tr><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>C</td><td>C</td></tr> <tr><td>C</td><td>D</td><td>D</td><td>D</td><td>D</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> </table>	B	B	B	B	B	B	B	B	C	C	C	D	D	D	D	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	B1	8 B's, 3 C's, 4 D's, 15 A's (but not just B C D A)	[5]															
B	B	B	B	B	B	B	B	C	C																																									
C	D	D	D	D	A	A	A	A	A																																									
A	A	A	A	A	A	A	A	A	A																																									
<table border="1"> <tr><td>Box 1</td><td>B</td><td>D</td><td>A</td><td>A</td></tr> <tr><td>Box 2</td><td>B</td><td>D</td><td>A</td><td>A</td></tr> <tr><td>Box 3</td><td>B</td><td>D</td><td>A</td><td>A</td></tr> <tr><td>Box 4</td><td>B</td><td>D</td><td>A</td><td>A</td></tr> <tr><td>Box 5</td><td>B</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr> <tr><td>Box 6</td><td>B</td><td>A</td><td></td><td></td></tr> <tr><td>Box 7</td><td>B</td><td></td><td></td><td></td></tr> <tr><td>Box 8</td><td>B</td><td></td><td></td><td></td></tr> <tr><td>Box 9</td><td>C</td><td>C</td><td>C</td><td></td></tr> </table>	Box 1	B	D	A	A	Box 2	B	D	A	A	Box 3	B	D	A	A	Box 4	B	D	A	A	Box 5	B	A	A	A	A	A	A	Box 6	B	A			Box 7	B				Box 8	B				Box 9	C	C	C		M1	Four boxes each containing B D A A (in any order)
Box 1	B	D	A	A																																														
Box 2	B	D	A	A																																														
Box 3	B	D	A	A																																														
Box 4	B	D	A	A																																														
Box 5	B	A	A	A	A	A	A																																											
Box 6	B	A																																																
Box 7	B																																																	
Box 8	B																																																	
Box 9	C	C	C																																															
<p>Box 5 is over the weight limit More than five A's is too heavy for one box</p>	M1	Using exactly 9 boxes, the first eight of which each contain a B (with or without other items) and the ninth contains three C's.																																																
	(iii)	Items may be the wrong shape for the boxes eg too tall	B1	Reference to shape, height, etc. but not practical issues connected with the food	[1]																																													
<b>Total = 11</b>																																																		

For reference				
Item type	A	B	C	D
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm <sup>3</sup> )	1 000	24 000	10 000	4 000
Weight (g)	1 000	250	300	400

5	(i)	<p>Minimise <math>2a - 3b + c + 18</math>  <math>\Rightarrow</math> minimise <math>2(20-x) - 3(10-y) + (8-z) + 18</math>  <math>\Rightarrow</math> minimise <math>-2x + 3y - z</math>  <math>\Rightarrow</math> maximise <math>2x - 3y + z</math> (given)</p> <p><math>a + b - c \geq 14</math>  <math>\Rightarrow (20-x) + (10-y) - (8-z) \geq 14</math>  <math>\Rightarrow x + y - z \leq 8</math> (given)</p> <p><math>-2a + 3c \leq 50</math>  <math>\Rightarrow -2(20-x) + 3(8-z) \leq 50</math>  <math>\Rightarrow 2x - 3z \leq 66</math> (given)</p> <p><math>10 + 4a \geq 5b</math>  <math>\Rightarrow 10 + 4(20-x) \geq 5(10-y)</math>  <math>\Rightarrow 4x - 5y \leq 40</math> (given)</p> <p><math>a \leq 20 \Rightarrow 20-x \leq 20 \Rightarrow x \geq 0</math>  <math>b \leq 10 \Rightarrow 10-y \leq 10 \Rightarrow y \geq 0</math>  <math>c \leq 8 \Rightarrow 8-z \leq 8 \Rightarrow z \geq 0</math></p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>(Constant has no effect on slope of objective)                  Replacing <math>a, b</math> and <math>c</math> in objective to get <math>-2x + 3y - z</math>                  (Condone omission of conversion to maximisation here)</p> <p>Replacing <math>a, b</math> and <math>c</math> in the first three constraints to get given expressions</p> <p>Showing how <math>a \leq 20, b \leq 10, c \leq 8</math> give <math>x \geq 0, y \geq 0, z \geq 0</math></p>	<p>[3]</p>																																																																																
(ii)		<table border="1" data-bbox="279 907 798 1064"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th><math>u</math></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>3</td> <td>-1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>-1</td> <td>1</td> <td>0</td> <td>0</td> <td>8</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>-3</td> <td>0</td> <td>1</td> <td>0</td> <td>66</td> </tr> <tr> <td>0</td> <td>4</td> <td>-5</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>40</td> </tr> </tbody> </table> <p><math>x</math> and <math>z</math> columns have negative entries in objective row, but <math>z</math> column has no positive entries in constraint rows, so pivot on <math>x</math> col  <math>8 \div 1 = 8; 66 \div 2 = 33; 40 \div 4 = 10</math>                  Least ratio is <math>8 \div 1</math>, so pivot on 1 from <math>x</math> col</p> <p>New row 2 = row 2                  New row 1 = row 1 + 2(new row 2)                  New row 3 = row 3 - 2(new row 2)                  New row 4 = row 4 - 4(new row 2)</p> <table border="1" data-bbox="279 1444 798 1601"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th><math>u</math></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>5</td> <td>-3</td> <td>2</td> <td>0</td> <td>0</td> <td>16</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>-1</td> <td>1</td> <td>0</td> <td>0</td> <td>8</td> </tr> <tr> <td>0</td> <td>0</td> <td>-2</td> <td>-1</td> <td>-2</td> <td>1</td> <td>0</td> <td>50</td> </tr> <tr> <td>0</td> <td>0</td> <td>-9</td> <td>4</td> <td>-4</td> <td>0</td> <td>1</td> <td>8</td> </tr> </tbody> </table> <p><math>x = 8, y = 0, z = 0</math></p> <p><math>x = 8 \Rightarrow a = 20 - 8 = 12</math>  <math>y = 0 \Rightarrow b = 10 - 0 = 10</math>  <math>z = 0 \Rightarrow c = 8 - 0 = 8</math></p>	$P$	$x$	$y$	$z$	$s$	$t$	$u$	RHS	1	-2	3	-1	0	0	0	0	0	1	1	-1	1	0	0	8	0	2	0	-3	0	1	0	66	0	4	-5	0	0	0	1	40	$P$	$x$	$y$	$z$	$s$	$t$	$u$	RHS	1	0	5	-3	2	0	0	16	0	1	1	-1	1	0	0	8	0	0	-2	-1	-2	1	0	50	0	0	-9	4	-4	0	1	8	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>Constraint rows correct, with three slack variable columns</p> <p>Objective row correct</p> <p>Choosing to pivot on <math>x</math> column (may be implied from pivot choice)</p> <p>Calculations seen or referred to and correct pivot choice made (cao)</p> <p>Pivot row unchanged (may be implied) or follow through for their +ve pivot</p> <p>Calculations for other rows shown (cao)</p> <p>An augmented tableau with three basis columns, non-negative values in final column and value of objective having not decreased</p> <p>Correct tableau after one iteration (cao)</p> <p>Non-negative values for <math>x, y</math> and <math>z</math> from their tableau</p> <p>Putting their values for <math>x, y</math> and <math>z</math> into <math>a = 20 - x, b = 10 - y</math> and <math>c = 8 - z</math></p> <p>Correct values for <math>a, b</math> and <math>c</math>, from their non-negative <math>x, y</math> and <math>z</math></p>	<p>[2]</p> <p>[2]</p> <p>[2]</p> <p>[2]</p> <p>[3]</p>
$P$	$x$	$y$	$z$	$s$	$t$	$u$	RHS																																																																														
1	-2	3	-1	0	0	0	0																																																																														
0	1	1	-1	1	0	0	8																																																																														
0	2	0	-3	0	1	0	66																																																																														
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0	0	-2	-1	-2	1	0	50																																																																														
0	0	-9	4	-4	0	1	8																																																																														
(iii)		<p><math>x \leq 20, y \leq 10</math> and <math>z \leq 8</math></p>	<p>M1</p> <p>A1</p>	<p>20, 10, 8</p> <p>Correct inequalities for <math>x, y</math> and <math>z</math></p>	<p>[2]</p>																																																																																
<p><b>Total = 16</b></p>																																																																																					

TO BE ANSWERED ON INSERT																																
6	(i)	10 $\frac{1}{2}n(n-1)$	B1 B1	10 $1+2+\dots+(n-1)$ seen, or equivalent Check that sum stops at $n-1$ not $n$	[2]																											
	(ii)(a)	9 1 2 3 45	B1 M1 A1	Their 10 minus 1 1, 2 and 3 45 following from method mark earned cao	[3]																											
	(b)	$1+2+3+\dots+(N-1)$ $= \frac{1}{2}N(N-1)$ , where $N = \frac{1}{2}n(n-1)$ $= \frac{1}{4}n(n-1)(\frac{1}{2}n(n-1) - 1)$ (given)	M1 A1	$1+2+3+\dots+(N-1)$ or $\frac{1}{2}N(N-1)$ , where $N = \frac{1}{2}n(n-1)$ Convincingly achieving the given result	[2]																											
	(iii)	<table border="1" style="display: inline-table; vertical-align: top;"> <thead> <tr> <th>M1 Vertices in tree</th> <th>M2 Arcs in tree</th> <th>M3 Vertices not in tree</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>ABCDE</td> </tr> <tr> <td>D E</td> <td>D   2   E</td> <td>A B C</td> </tr> <tr> <td>D E A</td> <td>D   2   E A   3   E</td> <td>B C</td> </tr> <tr> <td>D E A C</td> <td>D   2   E A   3   E A   4   C</td> <td>B</td> </tr> <tr> <td>DEACB</td> <td>D   2   E A   3   E A   4   C B   6   E</td> <td></td> </tr> </tbody> </table> <table border="1" style="display: inline-table; vertical-align: top; margin-left: 20px;"> <thead> <tr> <th>M4 Sorted list</th> </tr> </thead> <tbody> <tr><td><del>D   2   E</del></td></tr> <tr><td><del>A   3   E</del></td></tr> <tr><td><del>A   4   C</del></td></tr> <tr><td><del>C   5   D</del></td></tr> <tr><td><del>B   6   E</del></td></tr> <tr><td><del>B   7   C</del></td></tr> <tr><td><del>A   8   B</del></td></tr> <tr><td><del>C   9   E</del></td></tr> </tbody> </table>	M1 Vertices in tree	M2 Arcs in tree	M3 Vertices not in tree			ABCDE	D E	D   2   E	A B C	D E A	D   2   E A   3   E	B C	D E A C	D   2   E A   3   E A   4   C	B	DEACB	D   2   E A   3   E A   4   C B   6   E		M4 Sorted list	<del>D   2   E</del>	<del>A   3   E</del>	<del>A   4   C</del>	<del>C   5   D</del>	<del>B   6   E</del>	<del>B   7   C</del>	<del>A   8   B</del>	<del>C   9   E</del>		<p>(Order of entries in M1, M2 and M3 does not matter)</p> <p>M1 Arc <math>A   3   E</math> is added to M2, A is added to M1 and deleted from M3</p> <p>M1 Arc <math>A   4   C</math> is added to M2, C is added to M1 and deleted from M3</p> <p>M1 Arc <math>C   5   D</math> is not added to M2 and arc <math>B   6   E</math> is added to M2</p> <p>A1 cao (lists M1, M2 and M3 totally correct, ignore what is done in list M4).</p>	[4]
M1 Vertices in tree	M2 Arcs in tree	M3 Vertices not in tree																														
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<del>A   8   B</del>																																
<del>C   9   E</del>																																
	(iv)	$30 \times \left(\frac{500}{100}\right)^4$ $= 18750$ seconds	M1 A1	Or equivalent cao, with units (312 min 30 sec or 5 hours 12 min 30 sec)	[2]																											
<b>Total = 13</b>																																

# 4737 Decision Mathematics 2

<b>1</b>	<b>(i)</b>		B1	Bipartite graph correct	<b>[1]</b>			
	<b>(ii)</b>	 <p style="margin-left: 20px;"><math>D = T - C = G - B = S</math></p> <p style="margin-left: 20px;">Andy = food Beth = science Chelsey = geography Dean = television Elly = history</p>	B1	A new bipartite graph showing the pairings $AF, BG, CT$ and $EH$ but not $DS$	M1 This alternating path written down, not read off from labels on graph	A1 $B = S, C = G$ and $D = T$ written down	B1 $A = F, E = H$ written down	<b>[4]</b>
	<b>(iii)</b>	<p>Andy = food Beth = television Chelsey = geography Dean = politics Elly = history</p> <p>Science did not arise</p>	B1	$A = F, C = G, D = P$ and $E = H$ (cao) ( $B = T$ may be omitted)	B1	$S$ (cao)	<b>[2]</b>	
<b>Total = 7</b>								

2	<p>Add a dummy row</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>P</th> <th>R</th> <th>S</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>April</td> <td>30</td> <td>28</td> <td>32</td> <td>25</td> </tr> <tr> <td>May</td> <td>32</td> <td>34</td> <td>32</td> <td>35</td> </tr> <tr> <td>June</td> <td>40</td> <td>40</td> <td>39</td> <td>38</td> </tr> <tr> <td>Dummy</td> <td>40</td> <td>40</td> <td>40</td> <td>40</td> </tr> </tbody> </table> <p>Reduce rows</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>5</td> <td>3</td> <td>7</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Columns are already reduced</p> <p>Incomplete matching, cross through zeros</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>5</td> <td>3</td> <td>7</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Augment by 1</p> <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>4</td> <td>2</td> <td>6</td> <td>0</td> </tr> <tr> <td>0</td> <td>2</td> <td>0</td> <td>4</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>Complete matching</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>P</th> <th>R</th> <th>S</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>April</td> <td>4</td> <td>2</td> <td>6</td> <td>0</td> </tr> <tr> <td>May</td> <td>0</td> <td>2</td> <td>0</td> <td>4</td> </tr> <tr> <td>June</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>Dummy</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>April = Tall Trees                    £2500                      May = Palace                            £3200                      June = Sunnyside                    £3900</p> <p>Total cost = £9600</p>		P	R	S	T	April	30	28	32	25	May	32	34	32	35	June	40	40	39	38	Dummy	40	40	40	40	5	3	7	0	0	2	0	3	2	2	1	0	0	0	0	0	5	3	7	0	0	2	0	3	2	2	1	0	0	0	0	0	4	2	6	0	0	2	0	4	1	1	0	0	0	0	0	1		P	R	S	T	April	4	2	6	0	May	0	2	0	4	June	1	1	0	0	Dummy	0	0	0	1	<p>B1 Adding a dummy row of all equal values</p> <p>M1 Substantially correct attempt to reduce matrix (condone 1 numerical slip)</p> <p>A1 Correct reduced cost matrix from reducing rows first and statement of how table was formed, including reference to columns (cao)</p> <p>B1 Cross through zeros using minimum number of lines</p> <p>B1 Correct augmented matrix and statement of how table was formed (cao)</p> <p>B1 A = T, M = P, J = S (cao)</p> <p>B1 £9600 (cao) with units</p>	<p>[3]</p> <p>[2]</p> <p>[2]</p>
	P	R	S	T																																																																																																	
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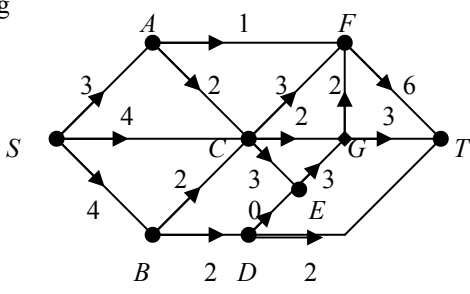
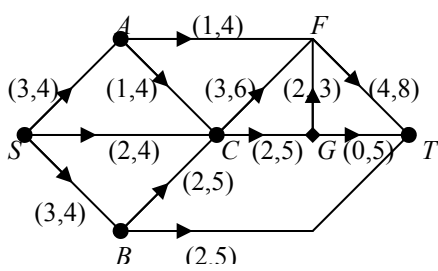
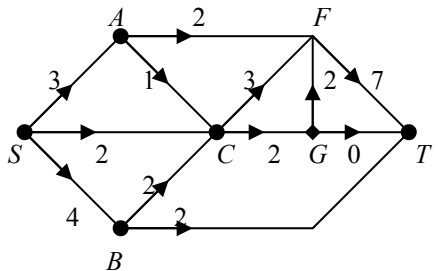


<p>3</p>	<p>(i)</p>		<p>Durations not necessary</p> <p>M1 Correct structure, even without directions shown Activities must be labelled</p> <p>A1 Completely correct, with exactly three dummies and all arcs directed</p>	<p>[2]</p>
	<p>(ii)</p>	<p>Minimum project completion time = 10 hours</p> <p>Critical activities <i>A, B, D, E, H</i></p>	<p>M1 Follow through their activity network if possible Substantially correct attempt at forward pass (at most 1 independent error)</p> <p>M1 Substantially correct attempt at backward pass (at most 1 independent error)</p> <p>A1ft Both passes wholly correct</p> <p>B1 10 hours (with units) cao</p> <p>M1 Either <i>B, E, H</i> or <i>A, D, H</i> (possibly with other critical activities, but <i>C, F, G</i> not listed). Not follow through.</p> <p>A1 <i>A, B, D, E, H</i> (and no others) cao</p>	<p>[3]</p> <p>[3]</p>
	<p>(iii)</p>	<p>No. of workers</p> <p>hours</p>	<p>On graph paper</p> <p>M1 A plausible resource histogram with no holes or overhangs</p> <p>A1 Axes scaled and labelled and histogram completely correct, cao</p>	<p>[2]</p>
	<p>(iv)</p>	<p>1 hour</p>	<p>B1 Accept 1 (with units missing) cao</p>	<p>[1]</p>
	<p>(v)</p>	<p>No need to change start times for <i>A, B, C, D</i> and <i>E</i> Activities <i>G</i> and <i>H</i> cannot happen at the same time, so they must follow one another This causes a 2 hour delay</p> <p><i>F</i> could be delayed until 1 hour before <i>H</i> starts <i>H</i> should be started as late as possible ⇒ a maximum delay of 3 hours</p>	<p>M1 <i>G</i> and <i>H</i> cannot happen together (stated, not just implied from a diagram)</p> <p>A1 2 cao</p> <p>B1 Diagram or explaining that for max delay on <i>F</i> need <i>H</i> to happen as late as possible</p> <p>B1 3 cao</p>	<p>[2]</p> <p>[2]</p>
<p style="text-align: right;"><b>Total = 15</b></p>				

4	(i)		B1	Correct structure (vertex labels and graph correct)	[3]																																							
			M1	Assigning weights to their graph (no more than 1 error or no more than 2 arcs missing/extra)																																								
			A1	Completely correct network																																								
	(ii)	Maximin	B1	cao	[1]																																							
	(iii)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Suboptimal maximin</th> </tr> </thead> <tbody> <tr> <td rowspan="3">2</td> <td rowspan="3">0</td> <td>0</td> <td>10</td> <td>10</td> </tr> <tr> <td>1</td> <td>10</td> <td>10</td> </tr> <tr> <td>2</td> <td>10</td> <td>10</td> </tr> <tr> <td rowspan="6">1</td> <td rowspan="3">0</td> <td>0</td> <td><math>\min(6,10) = 6</math></td> <td rowspan="6">8</td> </tr> <tr> <td>1</td> <td><math>\min(7,10) = 7</math></td> </tr> <tr> <td>2</td> <td><math>\min(8,10) = 8</math></td> </tr> <tr> <td rowspan="3">1</td> <td>0</td> <td><math>\min(6,10) = 6</math></td> </tr> <tr> <td>1</td> <td><math>\min(7,10) = 7</math></td> </tr> <tr> <td>2</td> <td><math>\min(8,10) = 8</math></td> </tr> <tr> <td rowspan="2">0</td> <td rowspan="2">0</td> <td>0</td> <td><math>\min(9,8) = 8</math></td> <td rowspan="2">8</td> </tr> <tr> <td>1</td> <td><math>\min(7,8) = 7</math></td> </tr> </tbody> </table> <p>Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)</p>	Stage	State	Action	Working	Suboptimal maximin	2	0	0	10	10	1	10	10	2	10	10	1	0	0	$\min(6,10) = 6$	8	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	1	0	$\min(6,10) = 6$	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	0	0	0	$\min(9,8) = 8$	8	1	$\min(7,8) = 7$	B1	Four or five columns, including 'stage', 'state' and 'action'	[3]
Stage	State	Action	Working	Suboptimal maximin																																								
2	0	0	10	10																																								
		1	10	10																																								
		2	10	10																																								
1	0	0	$\min(6,10) = 6$	8																																								
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0	0	0	$\min(9,8) = 8$	8																																								
		1	$\min(7,8) = 7$																																									
		B1	Stage and state columns completed correctly																																									
		B1	Action column completed correctly																																									
			M1	Min values correct for stage 1	[2]																																							
		A1	Suboptimal maximin values correct for stages 2 and 1 (follow through their network if possible, no more than 2 arcs missing/extra)																																									
			M1	Min values correct for stage 0	[2]																																							
		A1	Maximin value for stage 0 (follow through their network if possible, no more than 2 arcs missing/extra)																																									
			B1	8, cao	[2]																																							
			B1	Correct route, or in reverse																																								
<b>Total = 13</b>																																												

SR		Special ruling for working forwards																																															
	(iii)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Suboptimal maximin</th> </tr> </thead> <tbody> <tr> <td rowspan="2">1</td> <td rowspan="2">0</td> <td>0</td> <td>9</td> <td>9</td> </tr> <tr> <td>1</td> <td>7</td> <td>7</td> </tr> <tr> <td rowspan="6">2</td> <td rowspan="3">0</td> <td>0</td> <td><math>\min(9, 6) = 6</math></td> <td rowspan="6">6</td> </tr> <tr> <td>1</td> <td><math>\min(7, 6) = 6</math></td> </tr> <tr> <td>2</td> <td><math>\min(9, 7) = 7</math></td> </tr> <tr> <td rowspan="3">1</td> <td>0</td> <td><math>\min(9, 7) = 7</math></td> </tr> <tr> <td>1</td> <td><math>\min(7, 7) = 7</math></td> </tr> <tr> <td>2</td> <td><math>\min(9, 8) = 8</math></td> </tr> <tr> <td rowspan="2">2</td> <td rowspan="2">0</td> <td>0</td> <td><math>\min(9, 8) = 8</math></td> </tr> <tr> <td>1</td> <td><math>\min(7, 8) = 7</math></td> </tr> <tr> <td rowspan="3">3</td> <td rowspan="3">0</td> <td>0</td> <td><math>\min(6,10) = 6</math></td> <td rowspan="3">8</td> </tr> <tr> <td>1</td> <td><math>\min(7,10) = 7</math></td> </tr> <tr> <td>2</td> <td><math>\min(8,10) = 8</math></td> </tr> </tbody> </table> <p>Weight of heaviest truck = 8 tonnes Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)</p>	Stage	State	Action	Working	Suboptimal maximin	1	0	0	9	9	1	7	7	2	0	0	$\min(9, 6) = 6$	6	1	$\min(7, 6) = 6$	2	$\min(9, 7) = 7$	1	0	$\min(9, 7) = 7$	1	$\min(7, 7) = 7$	2	$\min(9, 8) = 8$	2	0	0	$\min(9, 8) = 8$	1	$\min(7, 8) = 7$	3	0	0	$\min(6,10) = 6$	8	1	$\min(7,10) = 7$	2	$\min(8,10) = 8$	B1	Four or five columns, including 'stage', 'state' and 'action'	[3]
Stage	State	Action	Working	Suboptimal maximin																																													
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		1	$\min(7,10) = 7$																																														
		2	$\min(8,10) = 8$																																														
		B0	No follow through from incorrect networks																																														
		B0																																															
			M1	Min values correct for stage 2 and suboptimal maximin values correct for stages 1 and 2 (cao)	[2]																																												
			A0																																														
			M1	No follow through from incorrect networks	[2]																																												
			A0	Correct min values for stage 3 and maximin value for stage 3 (cao)																																													
			B1	8, cao	[2]																																												
			B1	Correct route, or in reverse																																													
Maximum = B1 M1 M1 B1 B1 = 5 marks out of 9																																																	

<b>5</b>	<b>(i)</b>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td colspan="3" style="text-align: center;">Conan</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">G</td> <td style="text-align: center;">H</td> <td style="text-align: center;">I</td> <td style="text-align: center;">row min</td> </tr> <tr> <td></td> <td style="text-align: center;">D</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">-4</td> <td style="text-align: center;">2</td> <td style="text-align: center;">-4</td> </tr> <tr> <td style="text-align: center;">Robbie</td> <td style="text-align: center;">E</td> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-4</td> <td style="text-align: center;">-4</td> </tr> <tr> <td></td> <td style="text-align: center;">F</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">1</td> <td style="text-align: center;">-1</td> </tr> <tr> <td></td> <td style="text-align: center;">col max</td> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td></td> </tr> </table> <p style="text-align: center;">*</p>		Conan					G	H	I	row min		D	-1	-4	2	-4	Robbie	E	3	1	-4	-4		F	1	-1	1	-1		col max	3	1	2		<p>M1 Calculating row minima (cao)</p> <p>M1 Calculating column maxima (or their negatives) (cao)</p> <p>A1 Fairy or <i>F</i> (not just -1 or identifying row)</p> <p>A1 Hag or <i>H</i> (not just <math>\pm 1</math> or identifying column)</p> <p>B1 Follow through their play-safe for Conan Elf or <i>E</i></p>	[5]
		Conan																																				
		G	H	I	row min																																	
		D	-1	-4	2	-4																																
	Robbie	E	3	1	-4	-4																																
	F	1	-1	1	-1																																	
	col max	3	1	2																																		
<b>(ii)</b>	<p>Dwarf: <math>\frac{1}{3} [(-1) + (-4) + (2)] = -1</math></p> <p>Elf: <math>\frac{1}{3} [(3) + (1) + (-4)] = 0</math></p> <p>Fairy: <math>\frac{1}{3} [(1) + (-1) + (1)] = \frac{1}{3}</math></p>	<p>M1 <math>D = -1</math> or <math>F = \frac{1}{3}</math> or -3, 0, 1</p> <p>A1 All three correct</p>	[2]																																			
<b>(iii)</b>	<p>Goblin: <math>3p + (1-p) = 1 + 2p</math></p> <p>Hag: <math>p - (1-p) = 2p - 1</math></p> <p>Imp: <math>-4p + (1-p) = 1 - 5p</math></p> <p><math>2p - 1 = 1 - 5p</math></p> <p><math>\Rightarrow p = \frac{2}{7}</math></p>	<p>M1 Any one correct (in any form)</p> <p>A1 All three correct (in any form)</p> <p>M1 Appropriate equation seen for their expressions</p> <p>A1 <math>\frac{2}{7}</math> or 0.286 (or better) from method seen</p>	[2]																																			
<b>(iv)</b>	<p>4 is added throughout the table to make all the entries non-negative</p> <p>If Conan chooses the goblin, this gives an expected value (in the new table) of <math>3x + 7y + 5z</math></p>	<p>B1 Add 4 to remove negative values</p> <p>B1 Expected value when Conan chooses the goblin</p>	[2]																																			
<b>(v)</b>	<p><math>z = \frac{5}{7} \Rightarrow m \leq 5.571, m \leq 3.571, m \leq 3.571</math></p> <p><math>\Rightarrow m \leq 3.571 (3 \frac{4}{7}) (\frac{25}{7})</math></p> <p>Hence, maximum value for <i>M</i> is <math>3.571 - 4 = -0.429</math> or <math>-\frac{3}{7}</math></p>	<p>M1 Using <math>z = \frac{5}{7}</math> to find a value for <i>m</i> ( or implied)</p> <p>M1 Subtracting 4 from their <i>m</i> value</p> <p>A1 cao</p>	[3]																																			
<b>Total = 16</b>																																						

<b>6</b>	<b>(i)</b>	$\alpha = 12$ litres per second $\beta = 15$ litres per second	B1 B1	12 15	[2]
	<b>(ii)</b>	At least 3 litres per second must flow into $A$ , so $AC$ and $AF$ cannot both have flows of 1	B1	At least 3 flows along $SA$	[1]
	<b>(iii)</b>	At most 4 litres per second can flow into $B$ , and at least 4 must flow out, so $BC$ and $BD$ must have flows of 2  Hence, only 2 litres per second flows into $D$ and at least 2 litres per second must flow out, so $DE$ and $DT$ must both be at their lower capacities	B1  B1	At $B$ : flow in $\leq 4$ (and flow out $\geq 4$ ) hence given flows in $BC$ and $BD$  Stating that flow into $D$ is 2 and hence given flows in $DE$ and $DT$	[2]
	<b>(iv)</b>	Flow across $\{S, A, B, C\}$ , $\{D, E, F, G, T\} \geq 11$ (so 10 litres per second is impossible)  Minimum = 11 eg   Maximum = 12 No more than 12 can cross cut $\alpha$ and 12 is possible, eg augment flow shown above by 1 litre per second along $SAFT$	M1 A1  M1  A1  M1 A1	Or any equivalent reasoning (eg flow through $C$ ) Wholly convincing argument  11  Showing that 11 is possible (check $C$ )  12 Showing that 12 is possible but 13 is not	[2]  [2]  [2]
	<b>(v)</b>	  eg 	B1  M1 A1	A correct reduced network (vertex $E$ and all arcs incident on $E$ deleted), including arc capacities Or putting $E_{in}$ and $E_{out}$ with a capacity of 0 between them Or giving $CE$ , $EG$ and $DE$ upper and lower capacities of 0  On same diagram or a new diagram $SA = 3$ , $SC = 2$ , $SB = 4$ , $BC = 2$ and $BT = 2$ (and nothing through $E$ , if shown)  A valid flow of 9 litres per second through the network	[3]
<b>Total = 14</b>					

# Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)  
January 2010 Examination Series

## Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	51	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	52	44	36	28	21	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	51	44	37	30	24	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	62	53	44	35	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	47	40	34	28	22	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	51	45	39	33	28	0
	UMS	100	80	70	60	50	40	0

## Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
<b>3890</b>	300	240	210	180	150	120	0
<b>3891</b>	300	240	210	180	150	120	0
<b>3892</b>	300	240	210	180	150	120	0
<b>7890</b>	600	480	420	360	300	240	0
<b>7891</b>	600	480	420	360	300	240	0
<b>7892</b>	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
<b>3890</b>	28.2	53.1	73.0	87.2	96.4	100	1385
<b>3892</b>	39.2	61.7	79.2	92.5	97.5	100	126
<b>7890</b>	30.8	60.1	83.8	95.0	99.3	100	459
<b>7892</b>	21.1	60.5	84.2	100	100	100	43

For a description of how UMS marks are calculated see:

<http://www.ocr.org.uk/learners/ums/index.html>

Statistics are correct at the time of publication.

### List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF	Any equivalent form of answer or result is equally acceptable
AG	Answer given (working leading to the result must be valid)
CAO	Correct answer only
ISW	Ignore subsequent working
MR	Misread
SR	Special ruling
SC	Special case
ART	Allow rounding or truncating
CWO	Correct working only
SOI	Seen or implied
WWW	Without wrong working
Ft or $\checkmark$	Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

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