



Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

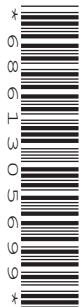
OCR supplied materials:

- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 A curve has equation $y = 2 + e^{\frac{1}{2}x}$. The region R is bounded by the curve and by the straight lines $x = 0$, $x = 4$ and $y = 0$. Find the exact volume of the solid obtained when R is rotated completely about the x -axis. [5]

- 2 (i) Use Simpson's rule with four strips to find an approximation to

$$\int_1^9 \ln x \ln(x+4) dx,$$

giving your answer correct to 4 significant figures. [4]

- (ii) Deduce an approximation to

$$\int_1^9 \ln(x^{-1}) \ln(x^2 + 8x + 16) dx,$$

giving your answer correct to 4 significant figures. [2]

- 3 (i) Sketch the graph of $y = |2x - 7a|$, where a is a positive constant. State the coordinates of the points where the graph meets each axis. [2]

- (ii) Solve the inequality $|2x - 7a| < 4a$. [3]

- (iii) Deduce the largest integer N satisfying the inequality $|2 \ln N - 10.5| < 6$. [2]

- 4 The angle θ , where $90^\circ < \theta < 180^\circ$, satisfies the equation

$$3 \sec^2 \theta + 10 \tan \theta = 11.$$

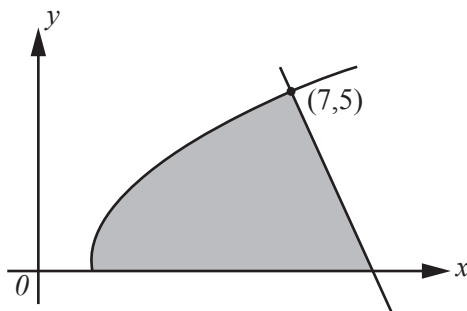
- (i) Find the value of $\tan \theta$. [3]

- (ii) Without using a calculator, determine the value of

- (a) $\tan 2\theta$, [2]

- (b) $\cot(2\theta + 135^\circ)$. [3]

5



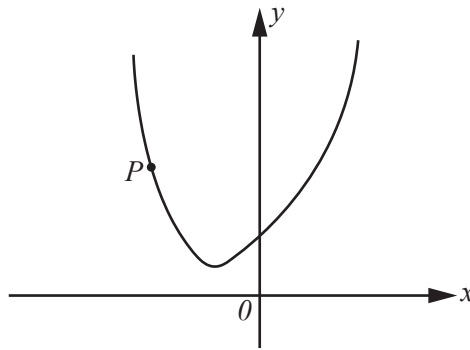
The diagram shows the curve $y = \sqrt{4x - 3}$ and the normal to the curve at the point $(7, 5)$. The shaded region is bounded by the curve, the normal and the x -axis. Find the exact area of the shaded region. [8]

- 6 (i) Give full details of a sequence of two transformations needed to transform the graph of $y = \frac{1}{x}$ to the graph of $y = \frac{3}{x+1}$. [2]

The function f is defined by $f(x) = \frac{3}{x+1}$ for $x \geq 0$.

- (ii) Determine the range of f . [2]
- (iii) Find an expression for $f^{-1}(x)$, and state how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. [3]
- (iv) Solve the equation $ff(x) = 2$. [3]
- 7 (i) It is given that $y = a^x$ where a is a positive constant. Express x in terms of $\ln y$ and, by first differentiating x with respect to y , show that $\frac{dy}{dx} = a^x \ln a$. [3]

(ii)



The diagram shows the curve $y = x^4 + 4^x$. At the point P on the curve, the gradient of the curve is -8 .

- (a) Show that the x -coordinate of P satisfies the equation $x = \sqrt[3]{-2 - 4^{x-1} \ln 4}$. [3]
- (b) By first using an iterative process based on the equation in part (a) with a starting value of -1 , find the coordinates of P . Show the result of each step of the iteration process and give the coordinates of P correct to 2 decimal places. [3]
- 8 (i) Express

$$3 \sin 2\theta \sec \theta + 4 \sin 2\theta \operatorname{cosec} \theta$$

in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [5]

(ii) Hence solve the equation

$$3 \sin(2\beta + 20^\circ) \sec(\beta + 10^\circ) + 4 \sin(2\beta + 20^\circ) \operatorname{cosec}(\beta + 10^\circ) = 3$$

for $0^\circ < \beta < 360^\circ$. [5]

- 9 (a) The equation of a curve has the form $y = \frac{px+q}{x^2+3}$. Show that the curve has two distinct stationary points for all non-zero values of the constants p and q . [4]
- (b) The equation of a curve has the form $y = e^{x^2}(ax^2+b)$, where a and b are non-zero constants. It is given that $\frac{d^2y}{dx^2}$ can be expressed in the form $e^{x^2}(cx^4+d)$, where c and d are non-zero constants. Prove that $5a+2b=0$. [5]

END OF QUESTION PAPER

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