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General Certificate of Education (A-level)
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Mathematics MPC2

(Specification 6360)

Pure Core 2

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	(common difference) = 9	B1	1	9
(b)	(100th term) = 23 + (100 − 1) d	M1		23 + (100 −1) d or better seen (or used with d = 9 or with d = c's answer (a))
	= 914	A1	2	914 NMS mark as B2 or B0
(c)	(Sum of series) = $\frac{280}{2}(23+2534)$ {or $\frac{280}{2}[2\times 23+(280-1)(9)]$ }	M1		Substitution of n = 280, l = 2534, a = 23 (or c's value of a used in (b)), d = 9 (or c's answer to (a)) into $\frac{n}{2}(a+l)$ PI or $\frac{n}{2}[2a+(n-1)d]$ PI
	= 357 980	A1	2	357 980 NMS mark as B2 or B0
	Total		**5**	
2(a)	(Area) = $\frac{1}{2}(26)(31.5)\sin\theta$	M1		$\frac{1}{2}(26)(31.5)\sin(\theta)$ stated or used
	$\frac{1}{2}(26)(31.5)\times\frac{5}{13}$ = 157.5 (cm^2)	A1	2	OE eg $\frac{315}{2}$ Condone AWRT 157.50 NMS: 157.5 or AWRT 157.50 scores B2
(b)	$(\cos\theta=)\frac{12}{13}$	B1	1	$\frac{12}{13}$ OE exact fraction
(c)	$\{AC^2=\}$ $31.5^2+26^2-2\times 31.5\times 26\times\cos(\theta)$	M1		RHS of cosine rule
	= 992.25 + 676 − 1512 = 1668.25 − 1512 = 156.25	m1		Correct order of evaluation. Do not award if evaluation leads to or would lead to RHS value being outside interval 120 to 195
	$AC=\sqrt{156.25}$ = 12.5 (cm)	A1	3	12.5 OE with no sight of premature approximation clearly used
	(Alternative) $\{AC^2=\}$ $(26\sin\theta)^2+(31.5-26\cos\theta)^2$ $=10^2+7.5^2$ $AC=\sqrt{156.25}$ = 12.5 (cm)	(M1) (m1) (A1)	(3)	
	Total		**6**	

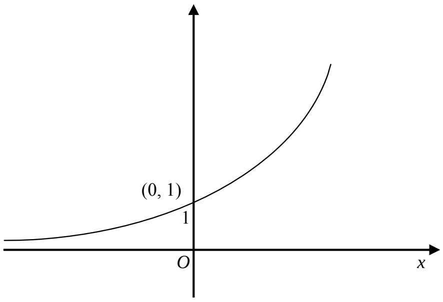
Q	Solution	Marks	Total	Comments
3(a)	$\ldots\ldots = \left(x^{\frac{3}{2}}\right)^2 - 2x^{\frac{3}{2}} + 1 = x^3 - 2x^{\frac{3}{2}} + 1$	B2,1,0	2	B2 for $x^3 - 2x^{\frac{3}{2}} + 1$ or $x^3 - 2x\sqrt{x} + 1$ (B1 fully correct unsimplified expression. seen eg $\left(x^{\frac{3}{2}}\right)^2 - x^{\frac{3}{2}} - x^{\frac{3}{2}} + 1$ or B1 for either $x^3 - 2x^{\frac{3}{2}} \ldots$ OE seen or $x^3 + 2x^{\frac{3}{2}} + 1$ OE seen or B1 for $-x^3 + 2x^{\frac{3}{2}} - 1$ OE seen)
(b)	$\int \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x = \frac{x^4}{4} - \frac{2x^{\frac{5}{2}}}{2.5} + x \ (+c)$	B1F		Ft on correct integration of all non $x^{\frac{3}{2}}$ terms (at least two) in c's expression. in (a)
		M1		Integration of a $kx^{\frac{3}{2}}$ as $\lambda x^{\frac{5}{2}}$ (ie power correct)
	$\{= 0.25x^4 - 0.8x^{2.5} + x\ (+c)\}$	A1F	3	Correct integration of c's $x^{\frac{3}{2}}$ term(s) ACF
(c)	$\int_1^4 \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$			
	$= \left(\frac{4^4}{4} - \frac{2(4^{\frac{5}{2}})}{2.5} + 4\right) - \left(\frac{1}{4} - \frac{2}{2.5} + 1\right)$	M1		F(4) − F(1) attempted following integration. If F(x) incorrect, ft c's answer to (b) provided integration attempted
	$\{= \frac{212}{5} - \frac{9}{20} = 42.4 - 0.45\} = 41.95$	A1	2	41.95 OE eg 839/20 Since 'Hence' NMS scores 0/2
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)	$u_1 = 12$	B1		CAO Must be 12
	$u_2 = 48 \times \frac{1}{16} = 3$	B1F	2	If not correct, ft on c's $u_1 \times \frac{1}{4}$
(b)	$r = \frac{1}{4}$	B1F	1	Only ft on r = (c's u_2)÷ (c's u_1) if $\lvert r \rvert<1$. Answers may be in equivalent fraction form or exact decimal form. If other notation used award the mark if correct or ft value confirmed in (c)
(c)	$(S_\infty =) \frac{u_1}{1-r} = \frac{12}{1-\frac{1}{4}}$	M1		Use of $\frac{a}{1-r}$, ft on c's u_1 and c's r in (a) and (b) if not recovered, provided $\lvert r \rvert < 1$
	= 16	A1F	2	If not 16, ft on c's u_1 and c's r in (a) and (b) provided $\lvert r \rvert < 1$.
(d)	$\sum_{n=4}^{\infty} u_n = S_\infty - \sum_{n=1}^{3} u_n$	M1		OE eg RHS $S_\infty - (u_1 + u_2 + u_3)$
	$u_3 = \frac{3}{4}$ (or $\sum_{n=1}^{3} u_n = \frac{12(1-0.25^3)}{1-0.25}$)	B1		Either result, or better eg $\sum_{n=1}^{3} u_n$ =15.75
	$\sum_{n=4}^{\infty} u_n = 0.25$	A1	3	NMS scores 0/3 **<u>SC</u>** For c's scoring 0/3 in (d); Award B1 to candidates who used $S_\infty - S_4$ for $\sum_{n=4}^{\infty} u_n$ and obtained the answer $\frac{1}{16}$ OE
	(Alternative)			
	$(\sum_{n=4}^{\infty} u_n = \frac{u_4}{1-r})$	(M1)		
	$(u_4 = \frac{3}{16}$ (= 0.1875))	(B1)		
	$(\sum_{n=4}^{\infty} u_n = \frac{3}{16} \div \frac{3}{4} = \frac{1}{4})$	(A1)	(3)	(NMS scores 0/3)
	Total		**8**	

Q	Solution	Marks	Total	Comments
5(a)	{Arc =} $r\theta$ $= 18\times\dfrac{2\pi}{3} = 12\pi$ (m)	M1 A1	 2	$r\theta$ seen or used for the arc length 12π
(b)(i)	$\alpha = \dfrac{\pi}{3}$	B1	1	$\dfrac{1}{3}\pi$ OE expression which simplifies to $\dfrac{1}{3}\pi$
(ii)	{Area of sector =} $\dfrac{1}{2}r^2\theta = \dfrac{1}{2}\times 18^2\times\dfrac{2\pi}{3}$	M1		$\dfrac{1}{2}r^2\theta$ seen or used for the sector area
	$= 108\pi$ (=339.(29..))	A1		If not exact accept 3sf or better PI by final correct answer
	$\tan\dfrac{\pi}{3} = \dfrac{TP}{18}$ {or $\tan\dfrac{\alpha}{2} = \dfrac{18}{TP}$} {or $PQ = 2\times 18\sin\dfrac{\pi}{3}$} {or $\dfrac{1}{2}PQ = 18\sin\dfrac{\pi}{3}$} $\left\{\text{or } \cos\dfrac{\pi}{3} = \dfrac{18}{OT}\right\}$ $\left\{\text{or } \sin\dfrac{\alpha}{2} = \dfrac{18}{OT}\right\}$	M1		OE Correct method (PI) to find either TP or TQ (=TP) or OT or PQ or $\dfrac{1}{2}PQ$. If α not $\pi/3$ then ft c's value for α in (b)(i). If c finds two of TP/TQ, OT and $PQ/\dfrac{1}{2}PQ$ and gets one correct, one wrong, mark correct one ie M1A1 (M1A0 possible if no correct length)
	$TP=18\sqrt{3}=31.1769\ldots$ exact or 31.1 to 31.2 incl} {or $PQ=18\sqrt{3}=31.1769...$ exact or 31.1 to 31.2 incl} {or $OT = 36$}; {$\dfrac{1}{2}PQ = 9\sqrt{3}$ or 15.5 to 15.6 incl}	A1		Correct TP or TQ or PQ or $\dfrac{1}{2}PQ$ or OT either exact value or in range indicated PI by value 561 to 561.3 inclusive for the area of the kite.
	Area of kite $PTQO = 2\times\dfrac{1}{2}\times 18\times TP$ {or Area $=\dfrac{1}{2}(18^2)\sin\dfrac{2\pi}{3}+\dfrac{1}{2}TP^2\sin\alpha$} {or area kite $=\dfrac{1}{2}\times PQ\times\left[18\div\cos\dfrac{\pi}{3}\right]$} {or area kite $=\dfrac{1}{2}\times 2\times 18\sin\dfrac{\pi}{3}\times OT$} {$=18^2\sqrt{3}$}{$= 2\times 162\sqrt{3}$};{$243\sqrt{3} + 81\sqrt{3}$}	M1		OE valid method to find area of kite, down to a correct expression with no more than 1 unknown length; ft on c's value of α. For method using > one unknown length this M is dependent on previous M for length PI by value $324\sqrt{3}$ or a numerical expression which simplifies to $324\sqrt{3}$; or a value 561 to 561.3 inclusive for the area of the kite. Can also be implied by award of the final A1
	Alternative Area triangle $PTQ = \dfrac{1}{2}TP^2\sin\alpha$ and Area triangle $POQ = \dfrac{1}{2}18^2\sin(2\pi/3)$	(M1)		OE Alternative: Award this method mark if **both** area of triangle PTQ (=$243\sqrt{3}$) and area of triangle POQ (=$81\sqrt{3}$) are found with or without finding area of kite
	Area of shaded region = $561.(18...) - 108\pi$ = 221.89… = 222 (m^2) to 3sf	A1	6	If not 222, condone value from 221.7 to 222.0 inclusive
	Alternative Area of shaded region = $243\sqrt{3} - (108\pi - 81\sqrt{3})$ = 221.89…= 222 (m^2) to 3sf	(A1)	(6)	
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)(i)	(When $x = 2$) $\frac{dy}{dx}$ = 12−1−11 = 0	B1	1	AG Must see intermediate evaluations
(ii)	$\frac{4}{x^2} = 4x^{-2}$ $\{$so $\frac{dy}{dx} = 3x^2 - 4x^{-2} - 11\}$	B1		$\frac{4}{x^2} = 4x^{-2}$, seen in (a)(ii) or earlier. PI by $\pm 8x^{-3}$ term in answer
	$\frac{d^2y}{dx^2} = 6x + 8x^{-3}$	M1		Correct powers of x correctly obtained from differentiating the first two terms
		A1		$6x + 8x^{-3}$ ACF
	When $x = 2$, $\frac{d^2y}{dx^2}$ = 12 + 8/8 = 13	A1	4	
(iii)	Since $\frac{d^2y}{dx^2} > 0$, P is a minimum point.	E1F	1	Ft on c's value of $y''(2)$ in (a)(ii) but must see reference to sign of $y''(2)$ either explicitly or as inequality, as well as the correct ft conclusion
(b)	$\int\left(3x^2 - \frac{4}{x^2} - 11\right)dx = x^3 + 4x^{-1} - 11x(+c)$	M1		Attempt to integrate $\frac{dy}{dx}$ with at least two of the three terms integrated correctly
	$(y =)\ x^3 + 4x^{-1} - 11x\ (+\ c)$	A1		For $x^3 + 4x^{-1} - 11x$ OE even unsimplified
	When $x = 2$, $y = 1 \Rightarrow 1 = 8 + 2 - 22 + c$	M1		Substituting. $x = 2$, $y = 1$ into $y = F(x)$ + 'c' in attempt to find constant of integration, where $F(x)$ follows attempted integration of expression for $\frac{dy}{dx}$
	$y = x^3 + 4x^{-1} - 11x + 13$	A1	4	ACF
	Total		**10**	

Q	Solution	Marks	Total	Comments
7(a)	$\tan\theta = -1$	B1		
	$\sin^2\theta = 3\cos^2\theta$			
	$\dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$	M1		$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ used on $\sin^2\theta - 3\cos^2\theta$ or forms and solves a correct quadratic in sin or cos and then uses to find $\tan\theta$
	$\tan^2\theta = 3$	A1		$\tan^2\theta = 3$ or $\tan^2\theta - 3 = 0$ or $\left(\tan\theta + \sqrt{3}\right)\left(\tan\theta - \sqrt{3}\right) = 0$ or $\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$
	$\tan\theta = \pm\sqrt{3}$	A1	4	Both
(b)	$\tan\theta = -1$, $\tan\theta = \sqrt{3}$, $\tan\theta = -\sqrt{3}$	M1		Uses part (a), at least as far as attempting to solve $\tan\theta = k$, where k is any one of c's values for $\tan\theta$
	(θ =) 135º, (θ =) 60º, (θ =) 120º	A2,1,0	3	If not A2 for all three correct, award A1 for two values correct **<u>Special Case</u>** If $\tan^2\theta = \dfrac{1}{3}$ in part (a) and M1 scored in (a) and in (b) then apply ft in part (b) ie A2F for θ= 135º, 30º, 150º. (A1F if two of these ft values) **<u>Special Case</u>**: If M0 then award B1 for any two correct values provided no incorrect extras in given interval. If > 3 answers in the given interval, deduct 1 mark for each extra in the given interval from any A marks awarded in (b). Ignore any answers outside $0 \le \theta \le 180$
	Total		**7**	

Q	Solution	Marks	Total	Comments
8(a)		B1		Correct shape, curve in 1st two quadrants only, crossing positive y-axis once and asymptotic to negative x-axis.
		B1	2	Coordinates (0, 1). Accept y-intercept indicated as 1 on diagram or stated as 'intercept = 1' B0 if graph clearly drawn crossing axes at more than one point
(b)(i)	$y^2 - 12 = y$ OE; $7^{2x} - 12 = 7^x$ OE	M1		Eliminates either x or y correctly
	$(y-4)(y+3)\ (=0)$; $(7^x - 4)(7^x + 3)\ (=0)$	A1		Correct factors or $y = \frac{1 \pm \sqrt{49}}{2}$ or better or $7^x = \frac{1 \pm \sqrt{49}}{2}$ or better
	Since $y\ (=7^x) > 0$, $[\,y\ (=7^x) \neq -3\,]$ (there is exactly one point of intersection)	E1		Clear indication that c's negative solution(s) has/have been considered and rejected
	y-coordinate is 4	B1	4	
(ii)	$7^x = 4$ so $x \log 7 = \log 4$ [or $x = \log_7 4$]	M1		OE ft on $7^x = k$, where k is positive, to either $x \log 7 = \log k$ or $x = \log_7 k$
	$x = 0.712(414\ldots) = 0.712$ to 3SF	A1	2	Condone > three significant figures. If use of logarithms not explicitly seen then score 0/2
	Total		**8**	

Q	Solution	Marks	Total	Comments
9(a)	$h = 0.25$ $f(x) = \log_{10}(x^2+1)$ $I \approx h/2\{\ldots\}$	B1		PI
	$\{.\} = f(0) + f(1) +$ $2\,[f(0.25) + f(0.5) + f(0.75)]$ $\{.\} =$	M1		OE summing of areas of the 'trapezia'
	$\log 1 + \log 2 + 2\left[\log\frac{17}{16} + \log\frac{5}{4} + \log\frac{25}{16}\right]$ $= 0 + 0.3010... +$ $2\,(0.0263... + 0.0969... + 0.1938...)$	A1		OE Accept 1sf evidence
	$= 0.3010...+2(0.317058...) = 0.935147\ldots$ $(I \approx)\ 0.125\,[0.935147\ldots] = 0.117$ (to 3SF)	A1	4	CAO Must be 0.117
(b)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	B1	1	
(c)(i)	$\log_{10}(10x^2) = \log_{10} 10 + \log_{10} x^2$	M1		Condone missing bases for M mark. Accept $\log x^2$ replaced by $2\log x$ in M1 line
	$= 1 + 2\log_{10} x$	A1	2	AG. Bases must be included or statement '$\log_{10} 10 = 1$' given.
				Condone missing bases in (c)(ii) & (c)(iii)
(ii)	$y = 1 + 2\log_{10} x = \log_{10}(10x^2)$	M1		PI
	Either $y = 2\log_{10}(\sqrt{10}\,x)$ (to compare $y = 2\log x$) or both $y = \log_{10} x^2$ and $y = \log_{10}(\sqrt{10}\,x)^2$	A1		Writing in correct form so that stretch details can be stated directly
	(Stretch) parallel to x-axis, sf $\frac{1}{\sqrt{10}}$ OE	B2,1,0	4	B2 for correct direction and scale factor ACF (B1 for correct exact scale factor ACF) (or B1 for 'x-direction, scale factor 1/10 ') (or B1for 'x-direction, scale factor $\sqrt{10}$ ') Apply ISW if dec follows exact values. (OE scale factor must be in exact form)
(iii)	$\log_{10}(10x^2) = \log_{10}(x^2+1)$	M1		PI by $10x^2 = x^2 + 1$ or correct x
	$(10x^2 = x^2 + 1,\ 9x^2 = 1$ and since $x > 0$) $x = \frac{1}{3}$	A1		$x = \frac{1}{3}$ OE stated or used; accept $\sqrt{\frac{1}{9}}, \frac{1}{\sqrt{9}}$
	(y-coordinate of P) $y = \log_{10}\frac{10}{9}$ Or $y = \log\left(\frac{1}{9}+1\right)$	A1		PI by $3\log\frac{10}{9}$ OE for the gradient of OP
	Gradient of OP = $3\log_{10}\frac{10}{9} = \log_{10}\frac{1000}{729}$	A1	4	$\log\frac{1000}{729}$; Accept 'a=1000, b=729'
	Total		**15**	
	TOTAL		**75**	