WJEC
CBAC

GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2014

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS C1-C4 & FP1-FP3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

1.	(*a*)	(i)	Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1
			Gradient of $AB = -{}^{1}/_{2}$ (or equivalent)	A1
		(ii)	A correct method for finding the equation of *AB* using the candidate's value for the gradient of *AB*.	M1
			Equation of AB : $y - 3 = -{}^{1}/_{2}\,(x - 12)$ (or equivalent) (f.t. the candidate's value for the gradient of *AB*)	A1
	(*b*)	(i)	Use of gradient $L \times$ gradient $AB = -1$	M1
			Equation of L : $y = 2x - 1$ (f.t. the candidate's value for the gradient of *AB*)	A1
		(ii)	A correct method for finding the coordinates of *D*	M1
			$D(4, 7)$ (convincing)	A1
		(iii)	A correct method for finding the length of *AD*(*BD*)	M1
			$AD = \sqrt{45}$	A1
			$BD = \sqrt{80}$	A1
	(*c*)	(i)	A correct method for finding the coordinates of *E*	M1
			$E(8, 15)$	A1
		(ii)	*ACBE* is a kite (c.a.o.)	B1

2.	(*a*)	$\dfrac{3\sqrt{3}+1}{5\sqrt{3}-7} = \dfrac{(3\sqrt{3}+1)(5\sqrt{3}+7)}{(5\sqrt{3}-7)(5\sqrt{3}+7)}$	M1
		Numerator: $45 + 21\sqrt{3} + 5\sqrt{3} + 7$	A1
		Denominator: $75 - 49$	A1
		$\dfrac{3\sqrt{3}+1}{5\sqrt{3}-7} = 2 + \sqrt{3}$ (c.a.o.)	A1
		Special case If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5\sqrt{3} - 7$	
	(*b*)	$\sqrt{12} \times \sqrt{24} = 12\sqrt{2}$	B1
		$\dfrac{\sqrt{150}}{\sqrt{3}} = 5\sqrt{2}$	B1
		$\dfrac{36}{\sqrt{2}} = 18\sqrt{2}$	B1
		$(\sqrt{12} \times \sqrt{24}) + \dfrac{\sqrt{150}}{\sqrt{3}} - \dfrac{36}{\sqrt{2}} = -\sqrt{2}$ (c.a.o.)	B1

3. (*a*) $\frac{dy}{dx} = 2x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 2 = -{}^{1}/_{4}(x - 6)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1

(*b*) Putting candidate's expression for $\frac{dy}{dx} = 2$ M1

x-coordinate of $Q = 5$ A1

y-coordinate of $Q = -1$ A1

$c = -11$ A1

(f.t. candidate's expression for $\frac{dy}{dx}$ and at most one error in the enumeration of the coordinates of Q for all three A marks provided both M1's are awarded)

4. (*a*) $(1 + x)^6 = 1 + 6x + 15x^2 + 20x^3 + \ldots$

All terms correct B2

If B2 not awarded, award B1 for three correct terms

(*b*) An attempt to substitute $x = 0{\cdot}1$ in the expansion of part (*a*)

(f.t. candidate's coefficients from part (*a*)) M1

$1{\cdot}1^6 \approx 1 + 6 \times 0{\cdot}1 + 15 \times 0{\cdot}01 + 20 \times 0{\cdot}001$

(At least three terms correct, f.t. candidate's coefficients from part (*a*))

A1

$1{\cdot}1^6 \approx 1{\cdot}77$ (c.a.o.) A1

5. (*a*) $a = 4$ B1

$b = -1$ B1

$c = 7$ B1

(*b*) An attempt to substitute 1 for x in an appropriate quadratic expression

(f.t. candidate's value for b) M1

Greatest value of $\frac{1}{4x^2 - 8x + 29} = \frac{1}{25}$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1

$b^2 - 4ac = (2k)^2 - 4 \times (k - 1) \times (7k - 4)$ — A1

Putting $b^2 - 4ac < 0$ — m1

$6k^2 - 11k + 4 > 0$ (convincing) — A1

Finding critical values $k = {}^1/_2$, $k = {}^4/_3$ — B1

A statement (mathematical or otherwise) to the effect that
$k < {}^1/_2$ or $k > {}^4/_3$ (or equivalent)
(f.t. candidate's derived critical values) — B2

Deduct 1 mark for each of the following errors
the use of non-strict inequalities
the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = -3(x + \delta x)^2 + 8(x + \delta x) - 7$ — B1

Subtracting y from above to find δy — M1

$\delta y = -6x\delta x - 3(\delta x)^2 + 8\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -6x + 8$$ (c.a.o.) A1

(*b*) $$\frac{dy}{dx} = 9 \times \frac{5}{4} \times x^{1/4} - 8 \times -\frac{1}{3} \times x^{-4/3}$$ B1, B1

8. **Either:** showing that $f(2) = 0$
Or: trying to find $f(r)$ for at least two values of r — M1

$f(2) = 0 \Rightarrow x - 2$ is a factor — A1

$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct — M1

$f(x) = (x - 2)(6x^2 - x - 2)$ — A1

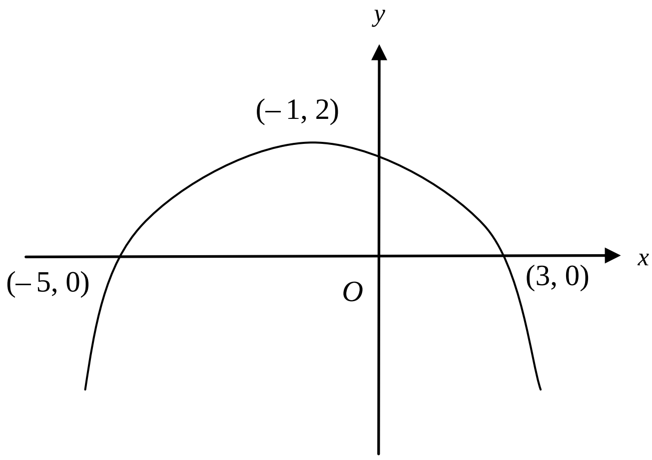
$f(x) = (x - 2)(3x - 2)(2x + 1)$ (f.t. only $6x^2 + x - 2$ in above line) — A1

$x = 2, {}^2/_3, -{}^1/_2$ (f.t. for factors $3x \pm 2$, $2x \pm 1$) — A1

Special case

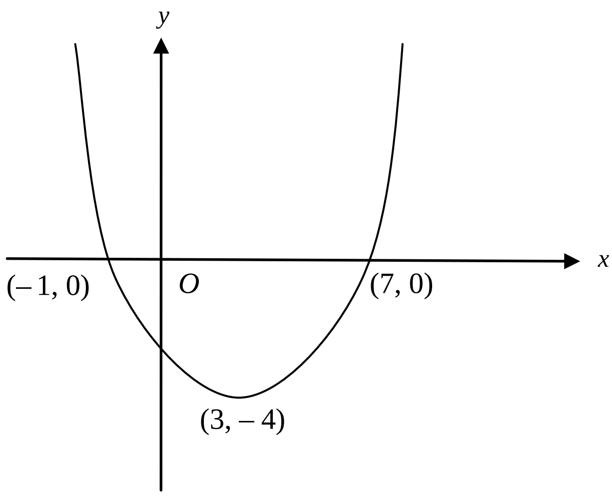
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

9. (*a*) (i)



Concave down curve with *y*-coordinate of maximum = 2	B1
x-coordinate of maximum = – 1	B1
Both points of intersection with *x*-axis	B1

(ii)



Concave up curve with *x*-coordinate of minimum = 3	B1
y-coordinate of minimum = – 4	B1
Both points of intersection with *x*-axis	B1

(*b*) $x = 3$ (c.a.o.) B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 18x + 27$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x + 3)^2 = 0 \Rightarrow x = -3$ (c.a.o) A1

$x = -3 \Rightarrow y = 4$ (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = -3^-$ and $x = -3^+$ M1

$\frac{dy}{dx}$ has same sign at $x = -3^-$ and $x = -3^+ \Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = -3$, $x = -3^-$ and $x = -3^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = -3$ and $\frac{d^2y}{dx^2}$ has different signs at $x = -3^-$ and $x = -3^+$

$\Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find the value of y at $x = -3^-$ and $x = -3^+$ M1

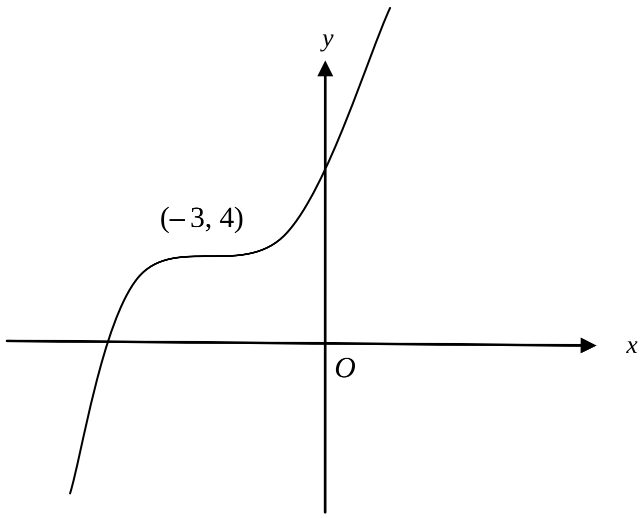
Value of y at $x = -3^- < 4$ and value of y at $x = -3^+ > 4 \Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = -3$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = -3 \Rightarrow (-3, 4)$ is a point of inflection A1

(*c*)



G1

1. (*a*)

1	0·301029995		
1·5	0·544068044		
2	0·698970004		
2·5	0·812913356		
3	0·903089987	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{2} \times \{0{\cdot}301029995 + 0{\cdot}903089987 + 2(0{\cdot}544068044 + 0{\cdot}698970004 + 0{\cdot}812913356)\}$

$I \approx 5{\cdot}31602279 \times 0{\cdot}5 \div 2$

$I \approx 1{\cdot}329005698$

$I \approx 1{\cdot}329$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

Special case for candidates who put $h = 0{\cdot}4$

1	0·301029995		
1·4	0·505149978		
1·8	0·643452676		
2·2	0·748188027		
2·6	0·832508912		
3	0·903089987	(all values correct)	B1

Correct formula with $h = 0{\cdot}4$ M1

$I \approx \frac{0{\cdot}4}{2} \times\{0{\cdot}301029995 + 0{\cdot}903089987 + 2(0{\cdot}505149978 + 0{\cdot}643452676 + 0{\cdot}748188027 + 0{\cdot}832508912))$

$I \approx 6{\cdot}662719168 \times 0{\cdot}4 \div 2$

$I \approx 1{\cdot}332543834$

$I \approx 1{\cdot}333$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*)

$\int_1^3 \log_{10}(3x - 1)^2\,dx \approx 2{\cdot}658$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $4\cos^2\theta + 1 = 4(1 - \cos^2\theta) - 2\cos\theta$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$8\cos^2\theta + 2\cos\theta - 3 = 0 \Rightarrow (2\cos\theta - 1)(4\cos\theta + 3) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$, $\cos\theta = -\frac{3}{4}$ (c.a.o.) A1

$\theta = 60°, 300°$ B1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks

$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $\alpha + 40° = 45°, 135°, \Rightarrow \alpha = 5°, 95°$ (at least one value of α) B1

$\alpha - 35° = 60°, 120°, \Rightarrow \alpha = 95°, 155°$ (at least one value of α) B1

$\alpha = 95°$ (c.a.o.) B1

(*c*) Correct use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ (o.e.) M1

$\tan\phi = \frac{10}{7}$ A1

$\phi = 55°, 235°$ (f.t $\tan\phi = a$) B1

3. (*a*) $\frac{y}{^4/_5} = \frac{x}{^8/_{17}}$ (o.e.) (correct use of sine rule) M1

$y = 1{\cdot}7x$ (convincing) A1

(*b*) $10{\cdot}5^2 = x^2 + y^2 - 2 \times x \times y \times (^{-13}/_{85})$

(correct use of the cosine rule) M1

Substituting $1{\cdot}7x$ for y in candidate's equation of form

$10{\cdot}5^2 = x^2 + y^2 \pm 2 \times x \times y \times {}^{13}/_{85}$ M1

$10{\cdot}5^2 = x^2 + 2{\cdot}89\,x^2 + 0{\cdot}52x^2$ (o.e.) A1

$x = 5$

(f.t. candidate's equation for x^2 provided both M's awarded) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{n}{2}[2 \times 3 + (n - 1) \times 2] = 360$ M1

Rewriting above equation in a form ready to be solved

$2n^2 + 4n - 720 = 0$ or $n^2 + 2n - 360 = 0$ or $n(n + 2) = 360$ A1

$n = 18$ (c.a.o.) A1

(*c*) $a + 9d = 7 \times (a + 2d)$ B1

$a + 7d + a + 8d = 80$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -5$, $d = 6$ (both values) (c.a.o.) A1

5. (*a*) $ar + ar^2 = -216$ B1

$ar^4 + ar^5 = 8$ B1

A correct method for solving the candidate's equations simultaneously

e,g multiplying the first equation by r^3 and subtracting

or eliminating a and $(1 + r)$ M1

$-216r^3 = 8$ (o.e.) A1

$r = -{}^1/_3$ (convincing) A1

(*b*) $a \times (-{}^1/_3) \times (1 - {}^1/_3) = -216 \Rightarrow a = 972$ B1

$S_\infty = \dfrac{972}{1 - (-{}^1/_3)}$ (correct use of formula for S_∞, f.t. candidate's derived value for a) M1

$S_\infty = 729$ (f.t. candidate's derived value for a) A1

6. (*a*) $5 \times \dfrac{x^{1/4}}{1/4} - 7 \times \dfrac{x^{3/2}}{3/2} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $16 - x^2 = x + 10$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant m1

$(x - 2)(x + 3) = 0 \Rightarrow x = 2, -3$ (both values, c.a.o.) A1

$y = 12, y = 7$ (both values, f.t. candidate's x-values) A1

(ii) Use of integration to find the area under the curve M1

$\int 16\,dx = 16x$, $\int x^2\,dx = (1/3)x^3$, (correct integration) B1

Correct method of substitution of candidate's limits m1

$[16x - (1/3)x^3]_{-3}^{2} = (32 - 8/3) - (-48 - (-9)) = 205/3$

Use of a correct method to find the area of the trapezium (f.t. candidate's coordinates for A, B) M1

Use of candidate's values for x_A and x_B as limits and trying to find total area by subtracting area of trapezium from area under curve m1

Shaded area = 205/3 – 95/2 = 125/6 (c.a.o.) A1

7. (*a*) **Either:**

$(5x/4 - 2)\log_{10}3 = \log_{10}7$

(taking logs on both sides and using the power law) M1

$\dfrac{5x}{4} = \dfrac{(\log_{10}7 + 2\log_{10}3)}{\log_{10}3}$ A1

$x = 3{\cdot}017$ (f.t. one slip, see below) A1

Or:

$5x/4 - 2 = \log_3 7$ (rewriting as a log equation) M1

$5x/4 = \log_3 7 + 2$ A1

$x = 3{\cdot}017$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}183$ from $\dfrac{5x}{4} = \dfrac{(\log_{10}7 - 2\log_{10}3)}{\log_{10}3}$

earns M1 A0 A1

an answer of $x = 0{\cdot}183$ from $\dfrac{5x}{4} = \dfrac{(2\log_{10}3 - \log_{10}7)}{\log_{10}3}$

earns M1 A0 A1

Note: Answer only with no working earns 0 marks

(*b*) (i) $b = a^5$ (relationship between log and power) B1

(ii) $a = b^{1/5}$ (the laws of indices) B1

$\log_b a = 1/5$ (relationship between log and power) B1

8. (*a*) (i) A correct method for finding the length of AB M1

$AB = 20$ A1

Sum of radii = distance between centres,

$\therefore$ circles touch A1

(ii) Gradient $AP(BP)(AB) = \dfrac{\text{inc in } y}{\text{inc in } x}$ M1

Gradient $AP = \dfrac{9-5}{-2-1} = -\dfrac{4}{3}$ (o.e) A1

Use of $m_{\tan} \times m_{\text{rad}} = -1$ M1

Equation of common tangent is:

$y - 5 = \dfrac{3}{4}(x-1)$ (o.e)

(f.t. one slip provided both M's are awarded) A1

(*b*) **Either:**

An attempt to rewrite the equation of C with l.h.s. in the form

$(x-a)^2 + (y-b)^2$ M1

$(x+2)^2 + (y-3)^2 = -7$ A1

Impossible, since r.h.s. must be positive ($= r^2$) A1

Or:

$g = 2$, $f = -3$, $c = 20$ and an attempt to use $r^2 = g^2 + f^2 - c$ M1

$r^2 = -7$ A1

Impossible, since r^2 must be positive A1

9. (*a*) (i) Area of sector $POQ = \frac{1}{2} \times r^2 \times 0{\cdot}9$ B1

(ii) Length of $PS = r \times \tan(0{\cdot}9)$ B1

(iii) Area of triangle $POS = \frac{1}{2} \times r \times r \times \tan(0{\cdot}9)$

(f.t. candidate's expression in r for the length of PS) B1

(*b*) $\frac{1}{2} \times r \times r \times \tan(0{\cdot}9) - \frac{1}{2} \times r^2 \times 0{\cdot}9 = 95{\cdot}22$

(f.t. candidate's expressions for area of sector and area of triangle, at least one correct) M1

$r^2 = \dfrac{2 \times 95{\cdot}22}{(1{\cdot}26 - 0{\cdot}9)}$ (o.e.) (c.a.o.) A1

$r = 23$ (f.t. one numerical slip) A1

1. (*a*)

0	2·197224577		
0·75	2·314217179		
1·5	2·524262696		
2·25	2·861499826		
3	3·335254744	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}75$ M1

$I \approx \frac{0{\cdot}75}{3} \times \{2{\cdot}197224577 + 3{\cdot}335254744 + 4(2{\cdot}314217179 + 2{\cdot}861499826) + 2(2{\cdot}524262696)\}$

$I \approx 31{\cdot}28387273 \times 0{\cdot}75 \div 3$

$I \approx 7{\cdot}820968183$

$I \approx 7{\cdot}82$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^3 \ln(16 + 2e^x)\,dx = \int_0^3 \ln(8 + e^x)\,dx + \int_0^3 \ln 2\,dx$ M1

$\int_0^3 \ln(16 + 2e^x)\,dx = 7{\cdot}82 + 2{\cdot}08 = 9{\cdot}90$

(f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. $8(\sec^2\theta - 1) - 5\sec^2\theta = 7 + 4\sec\theta.$ (correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant m1

$3\sec^2\theta - 4\sec\theta - 15 = 0 \Rightarrow (3\sec\theta + 5)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = -\frac{5}{3}, \sec\theta = 3$

$\Rightarrow \cos\theta = -\frac{3}{5}, \cos\theta = \frac{1}{3}$ (c.a.o.) A1

$\theta = 126{\cdot}87°, 233{\cdot}13°$ B1 B1

$\theta = 70{\cdot}53°, 289{\cdot}47°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{d}{dx}(8xy^2) = (8x)(2y)\frac{dy}{dx} + 8y^2$ B1

$\frac{d}{dx}(2x^2) = 4x, \ \frac{d}{dx}(9) = 0$ B1

$\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$ (convincing) (c.a.o.) B1

(*b*) $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$ B1

Substitute $2y^2$ for x in equation of C M1

$9y^4 + 9 = 0$ (o.e.) (c.a.o.) A1

$9y^4 + 9 > 0$ for any real y (o.e.) and thus no such point exists A1

4. candidate's x-derivative $= 2e^t$ B1

candidate's y-derivative $= -8e^{-t} + 3e^t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{-8e^{-t} + 3e^t}{2e^t}$ (o.e.) (c.a.o.) A1

Putting candidate's $\frac{dy}{dx} = -1$, rearranging and obtaining either an equation in **both** e^t and e^{-t}, or an equation in e^{2t}, or an equation in e^{-2t}. M1

Either $e^{2t} = \frac{8}{5}$ or $e^{-2t} = \frac{5}{8}$

(f.t. one numerical slip in candidate's derived expression for $\frac{dy}{dx}$) A1

$t = 0{\cdot}235$ (c.a.o.) A1

5. (*a*) $\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{ax + b}{3x^2 - 2x - 1}$ (including $a = 0$, $b = 1$) M1

$\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{6x - 2}{3x^2 - 2x - 1}$ A1

$6x - 2 = 8x(3x^2 - 2x - 1)$ (o.e.) (f.t. candidate's a, b) A1

$12x^3 - 8x^2 - 7x + 1 = 0$ (convincing) A1

(*b*) $x_0 = -0{\cdot}6$

$x_1 = -0{\cdot}578232165$ (x_1 correct, at least 4 places after the point) B1

$x_2 = -0{\cdot}582586354$

$x_3 = -0{\cdot}581770386$

$x_4 = -0{\cdot}581925366 = -0{\cdot}5819$ (x_4 correct to 4 decimal places) B1

Let $g(x) = 12x^3 - 8x^2 - 7x + 1$

An attempt to check values or signs of $g(x)$ at $x = -0{\cdot}58185$, $x = -0{\cdot}58195$ M1

$g(-0{\cdot}58185) = 7{\cdot}35 \times 10^{-4}$, $g(-0{\cdot}58195) = -7{\cdot}15 \times 10^{-4}$ A1

Change of sign $\Rightarrow \alpha = -0{\cdot}5819$ correct to four decimal places A1

6. (*a*) (i) $\dfrac{dy}{dx} = -\dfrac{1}{4} \times (9 - 4x^5)^{-5/4} \times f(x)$ $(f(x) \neq 1)$ M1

$\dfrac{dy}{dx} = -\dfrac{1}{4} \times (9 - 4x^5)^{-5/4} \times (-20x^4)$

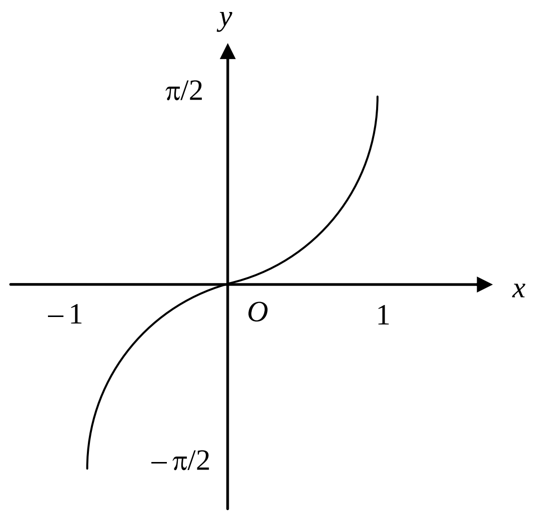
$\dfrac{dy}{dx} = 5x^4 \times (9 - 4x^5)^{-5/4}$ A1

(ii) $\dfrac{dy}{dx} = \dfrac{(7 - x^3) \times f(x) - (3 + 2x^3) \times g(x)}{(7 - x^3)^2}$ $(f(x), g(x) \neq 1)$ M1

$\dfrac{dy}{dx} = \dfrac{(7 - x^3) \times 6x^2 - (3 + 2x^3) \times (-3x^2)}{(7 - x^3)^2}$ A1

$\dfrac{dy}{dx} = \dfrac{51x^2}{(7 - x^3)^2}$ (c.a.o.) A1

(*b*) (i)



G1

(ii) $x = \sin y \Rightarrow \dfrac{dx}{dy} = \cos y$ B1

$\dfrac{dx}{dy} = \pm\sqrt{(1 - \sin^2 y)}$ B1

The +ive sign is chosen because the graph shows the gradient to be positive E1

$\dfrac{dx}{dy} = \sqrt{(1 - x^2)}$ B1

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{(1 - x^2)}}$ B1

7. (*a*) (i) $\int \cos(2-5x)\,dx = k \times \sin(2-5x) + c$ ($k = 1, {}^1/_5, -5, -\mathbf{{}^1/_5}$) M1

$\int \cos(2-5x)\,dx = -{}^1/_5 \times \sin(2-5x) + c$ A1

(ii) $\int \frac{4}{e^{3x-2}}\,dx = k \times 4 \times e^{2-3x} + c$ ($k = 1, -3, {}^1/_3\ \mathbf{-{}^1/_3}$) M1

$\int \frac{4}{e^{3x-2}}\,dx = -{}^4/_3 \times e^{2-3x} + c$ A1

(iii) $\int \frac{5}{{}^1/_6 x - 3}\,dx = k \times 5 \times \ln|{}^1/_6 x - 3| + c$ ($k = 1, {}^1/_6, \mathbf{6}$) M1

$\int \frac{5}{{}^1/_6 x - 3}\,dx = 30 \times \ln|{}^1/_6 x - 3| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (4x+1)^{1/2}\,dx = k \times \frac{(4x+1)^{3/2}}{3/2}$ ($k = 1, 4, \mathbf{{}^1/_4}$) M1

$\int_2^6 (4x+1)^{1/2}\,dx = \left[{}^1/_4 \times \frac{(4x+1)^{3/2}}{3/2}\right]_2^6$ A1

A correct method for substitution of limits in an expression of the form $m \times (4x+1)^{3/2}$ M1

$\int_2^6 (4x+1)^{1/2}\,dx = \frac{125}{6} - \frac{27}{6} = \frac{98}{6} = 16{\cdot}33$

(f.t. only for solutions of $\frac{392}{6}$ and $\frac{1568}{6}$ from $k = 1, 4$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Choice of *a*, *b*, with one positive and one negative and one side correctly evaluated M1

Both sides of identity evaluated correctly A1

(*b*) Trying to solve $3x - 2 = 7x$ M1

Trying to solve $3x - 2 = -7x$ M1

$x = -0{\cdot}5,\ x = 0{\cdot}2$ (both values) (c.a.o.) A1

Alternative mark scheme

$(3x-2)^2 = 7^2 \times x^2$ (squaring both sides) M1

$40x^2 + 12x - 4 = 0$ (o.e.) (c.a.o.) A1

$x = -0{\cdot}5,\ x = 0{\cdot}2$ (both values, f.t. one slip in quadratic) A1

9. (*a*) $f(x) = (x - 4)^2 - 9$ B1

(*b*) $y = (x - 4)^2 - 9$ and an attempt to isolate x
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) M1
$x = (\pm)\sqrt{(y + 9)} + 4$
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
$x = -\sqrt{(y + 9)} + 4$ (o.e.) (c.a.o.) A1
$f^{-1}(x) = -\sqrt{(x + 9)} + 4$ (o.e.)
(f.t. only incorrect choice of sign in front of the $\sqrt{}$ sign and candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1

10. (*a*) $R(g) = [2k - 4, \infty)$ B1

(*b*) (i) $2k - 4 \geq -2$ M1
$k \geq 1$ ($\Rightarrow$ least value of k is 1)
(f.t. candidate's $R(g)$ provided it is of form $[a, \infty)$) A1

(ii) $fg(x) = (kx - 4)^2 + k(kx - 4) - 8$ B1

(iii) $(3k - 4)^2 + k(3k - 4) - 8 = 0$
(substituting 3 for x in candidate's expression for $fg(x)$ and putting equal to 0) M1
Either $12k^2 - 28k + 8 = 0$ or $6k^2 - 14k + 4 = 0$
or $3k^2 - 7k + 2 = 0$ (c.a.o.) A1
$k = {}^1/_3, 2$ (f.t. candidate's quadratic in k) A1
$k = 2$ (c.a.o.) A1

1. $9x^2 - 5x \times 2y\frac{dy}{dx} - 5y^2 + 8y^3\frac{dy}{dx} = 0$ $\left(-5x \times 2y\frac{dy}{dx} - 5y^2\right)$ B1

$\left(9x^2 + 8y^3\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

2. (*a*) $f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-4)}$ (correct form) M1

$5x^2 + 7x + 17 \equiv A(x-4) + B(x+1)(x-4) + C(x+1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3$, $C = 5$, $B = 0$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2}{(x+1)^2}$ M1

$\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{-1}{(x+1)^2} + \frac{5}{(x-4)}$

(f.t. candidates values for A, B, C) A1

3. (*a*) $\dfrac{2\tan x}{1-\tan^2 x} = 3\cot x$ (correct use of formula for $\tan 2x$) M1

$\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{3}{\tan x}$ (correct use of $\cot x = \dfrac{1}{\tan x}$) M1

$\tan^2 x = {}^3/_5$ (o.e.) A1

$x = 37{\cdot}76°, 142{\cdot}24°$ (both values)

(f.t. $a\tan^2 x = b$ provided both M1's are awarded) A1

(*b*) (i) $R = 29$ B1

Correctly expanding $\sin(\theta - \alpha)$ and using either $29\cos\alpha = 21$ **or** $29\sin\alpha = 20$ **or** $\tan\alpha = \dfrac{20}{21}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 43{\cdot}6°$ (c.a.o) A1

(ii) Greatest value of $\dfrac{1}{21\sin\theta - 20\cos\theta + 31} = \dfrac{1}{29\times(\pm 1) + 31}$

(f.t. candidate's value for R) M1

Greatest value $= \dfrac{1}{2}$ (f.t. candidate's value for R) A1

Corresponding value for $\theta = 313{\cdot}6°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume $= \pi \displaystyle\int_0^{\pi/4} (3 + 2\sin x)^2\,dx$ B1

Correct use of $\sin^2 x = \dfrac{(1-\cos 2x)}{2}$ M1

Integrand $= (9 + 2 + 12\sin x - 2\cos 2x)$ (c.a.o.) A1

$\displaystyle\int (a + b\sin x + c\cos 2x)\,dx = (ax - b\cos x + \frac{c}{2}\sin 2x)$ $(a \ne 0, b \ne 0, c \ne 0)$ B1

Correct substitution of correct limits in candidate's integrated expression of form $(ax - b\cos x + \frac{c}{2}\sin 2x)$ $(a \ne 0, c \ne 0)$ M1

Volume $= 35$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. $(1-2x)^{1/2} = 1 + (1/2) \times (-2x) + \dfrac{(1/2) \times (1/2 - 1) \times (-2x)^2}{1 \times 2} + \ldots$

(– 1 each incorrect term) B2

$\dfrac{1}{1+4x} = 1 + (-1) \times (4x) + \dfrac{(-1) \times (-2) \times (4x)^2}{1 \times 2} + \ldots$

(– 1 each incorrect term) B2

$6\sqrt{1-2x} - \dfrac{1}{1+4x} = 5 - 2x - 19x^2 + \ldots$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/4$ (o.e.) B1

6. (*a*) candidate's x-derivative = 2

candidate's y-derivative = $15t^2$ (at least one term correct)

and use of

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{15}{2}t^2$ (o.e.) (c.a.o.) A1

Equation of tangent at P: $y - 5p^3 = \dfrac{15}{2}p^2(x - 2p)$

(f.t. candidate's expression for $\dfrac{dy}{dx}$) m1

$2y = 15p^2x - 20p^3$ (convincing) A1

(*b*) Substituting $p = 1$, $x = 2q$, $y = 5q^3$ in equation of tangent M1

$q^3 - 3q + 2 = 0$ (convincing) A1

Putting $f(q) = q^3 - 3q + 2$

Either $f(q) = (q-1)(q^2 + q - 2)$ **or** $f(q) = (q+2)(q^2 - 2q + 1)$ M1

Either $f(q) = (q-1)(q-1)(q+2)$ **or** $q = 1$, $q = -2$ A1

$q = -2$ A1

7. (*a*) $u = \ln 2x \Rightarrow \mathrm{d}u = 2 \times \dfrac{1}{2x}\,\mathrm{d}x$ (o.e.) B1

$\mathrm{d}v = x^4\,\mathrm{d}x \Rightarrow v = \dfrac{1}{5}x^5$ (o.e.) B1

$\int x^4 \ln 2x\,\mathrm{d}x = \ln 2x \times \dfrac{1}{5}x^5 - \int \dfrac{1}{5}x^5 \times \dfrac{1}{x}\,\mathrm{d}x$ (o.e.) M1

$\int x^4 \ln 2x\,\mathrm{d}x = \ln 2x \times \dfrac{1}{5}x^5 - \dfrac{1}{25}x^5 + c$ (c.a.o.) A1

(*b*) $\int \sqrt{(10\cos x - 1)}\sin x\,\mathrm{d}x = \int k \times u^{1/2}\,\mathrm{d}u$ $(k = -{}^{\mathbf{1}}/_{\mathbf{10}},\ {}^{1}/_{10} \text{ or } \pm 10)$ M1

$\int a \times u^{1/2}\,\mathrm{d}u = a \times \dfrac{u^{3/2}}{3/2}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)}\sin x\,\mathrm{d}x = k\left[\dfrac{u^{3/2}}{3/2}\right]_9^4$ or $k\left[\dfrac{(10\cos x - 1)^{3/2}}{3/2}\right]_0^{\pi/3}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)}\sin x\,\mathrm{d}x = \dfrac{19}{15} = 1{\cdot}27$ (c.a.o.) A1

8. (*a*) $\dfrac{\mathrm{d}V}{\mathrm{d}t} = kV$ B1

(*b*) $\int \dfrac{\mathrm{d}V}{V} = \int k\,\mathrm{d}t$ M1

$\ln V = kt + c$ A1

$V = \mathrm{e}^{kt + c} = A\mathrm{e}^{kt}$ (convincing) A1

(*c*) (i) $292 = A\mathrm{e}^{2k}$

$637 = A\mathrm{e}^{28k}$ (both values) B1

Dividing to eliminate A M1

$\dfrac{637}{292} = \mathrm{e}^{26k}$ A1

$k = \dfrac{1}{26}\ln\left[\dfrac{637}{292}\right] = 0{\cdot}03$ A1

(ii) $A = 275$ B1

(iii) When $t = 0$, initial value of investment = £275

(f.t. candidate's derived value for A) B1

9. (*a*) $\mathbf{p.q} = -18$ B1

$|\mathbf{p}| = \sqrt{14}$, $|\mathbf{q}| = \sqrt{105}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p.q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 118°$ (c.a.o.) A1

(*b*) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and

$\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$

(convincing) B1

Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of *CD* M1

Vector equation of *CD*: $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1-\lambda)\mathbf{b}$ (convincing) A1

(ii) **Either:**

Either substituting $\frac{1}{3}$ for λ in the vector equation of *CD*

or substituting 2 for μ in the vector equation of *L* M1

At least one of these position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1

Both position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position vector of the point of intersection *E* A1

Or:

$2\lambda = \frac{\mu}{3}$

$\frac{1}{2}(1-\lambda) = \frac{1}{3}(\mu - 1)$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{1}{3}$ or $\mu = 2$ A1

$\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1

(iii) **Either**: *E* lies on *AB* and is such that $AE : EB = 1 : 2$ (o.e.)

Or: *E* is the point of intersection of *AB* and *CD* B1

10. Squaring both sides we have

$1 + 2\sin\theta\cos\theta > 2$ B1

$\sin 2\theta > 1$ B1

Contradiction, since the sine of any angle ≤ 1 B1

Ques	Solution	Mark	Notes
1(a)	$f(x+h)-f(x)=\frac{1}{(x+h)^2}-\frac{1}{x^2}$	M1A1	
	$=\frac{x^2-(x+h)^2}{x^2(x+h)^2}$	A1	
	$=\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}$		
	$=\frac{-2xh-h^2}{x^2(x+h)^2}$	A1	
	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	M1	
	$=\lim_{h\to 0}\frac{-2xh-h^2}{hx^2(x+h)^2}=-\frac{2}{x^3}$	A1	
(b)			
	$\ln f(x)=x\ln\sec x$	B1	
	$\frac{f'(x)}{f(x)}=\ln\sec x+\frac{x\sec x\tan x}{\sec x}$	B1B1	B1 each side
	$f'(x)=(\sec x)^x(\ln\sec x+x\tan x)$	B1	
2(a)	$S_n=\sum_{r=1}^{n}r(r+3)=\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}3r$	M1	
	$=\frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}$	A1	
	$=\frac{n(n+1)}{6}(2n+1+9)$	m1	
	$=\frac{n(n+1)(n+5)}{3}$ or $\frac{n^3+6n^2+5n}{3}$ oe	A1	
(b)			
	$T_n=S_n-S_{n-1}$	M1	
	$=n(n+3)-(n-1)(n+2)$	A1	
	$=n^2+3n-(n^2+n-2)$		
	$=2(n+1)$	A1	

Ques	Solution	Mark	Notes
3(a)	$x + 2y + 4z = 3$ $x - y + 2z = 4$ $4x - y + 10z = k$		
	Attempting to use row operations,	M1	
	$x + 2y + 4z = 3$ $3y + 2z = -1$	A1	
	$9y + 6z = 12 - k$	A1	
	Since the 3rd equation is three times the 2nd equation, it follows that	M1	
	$12 - k = -3$; $k = 15$	A1	
(b)	Let $z = \alpha$	M1	
	$y = -\dfrac{(1+2\alpha)}{3}$	A1	
	$x = \dfrac{11-8\alpha}{3}$ (or equivalent)	A1	
4	EITHER $z = \dfrac{1+2\mathrm{i}}{1-\mathrm{i}} \times \dfrac{1+\mathrm{i}}{1+\mathrm{i}}$	M1	
	$= \dfrac{1+2\mathrm{i}+\mathrm{i}+2\mathrm{i}^2}{1-\mathrm{i}+\mathrm{i}-\mathrm{i}^2)}$	A1	
	$= \dfrac{-1+3\mathrm{i}}{2}$	A1	
	Mod(z) = $\dfrac{\sqrt{10}}{2}$ $(\sqrt{2.5}, 1.58)$	B1	FT their z
	Arg(z) = $\tan^{-1}(-3) + \pi$ $= 1.89$ $(108°)$	M1A1	Award M1A0 for $\tan^{-1}(-3)$ (– 1.25 or – 72°)
	OR		
	Mod(1 + 2i) $= \sqrt{5}$	B1	
	Mod(1 – i) = $\sqrt{2}$	B1	
	Mod$\left(\dfrac{1+2\mathrm{i}}{1-\mathrm{i}}\right) = \sqrt{\dfrac{5}{2}}$	B1	
	Arg(1 + 2i) = $\tan^{-1} 2 = 1.107..$	B1	FT one incorrect mod
	Arg(1 – i) = $\tan^{-1}(-1) = -0.785..$	B1	
	Arg$\left(\dfrac{1+2\mathrm{i}}{1-\mathrm{i}}\right) = 1.107.. + 0.785..$ $= 1.89$ $(108°)$	B1	FT one incorrect arg

Ques	Solution	Mark	Notes
5(a)	$\alpha+\beta+\gamma=-2,\ \beta\gamma+\gamma\alpha+\alpha\beta=2,\ \alpha\beta\gamma=-3$	B1	
	$\beta\gamma\times\gamma\alpha+\beta\gamma\times\alpha\beta+\gamma\alpha\times\alpha\beta=\alpha\beta\gamma(\alpha+\beta+\gamma)$	M1	FT their first line if one error
	$=-3\times-2=6$	A1	
	$\beta\gamma\times\gamma\alpha\times\alpha\beta=(\alpha\beta\gamma)^2=9$	M1A1	
	The required equation is $x^3-2x^2+6x-9=0$	B1	FT previous values
(b)	$\alpha^2+\beta^2+\gamma^2$		
	$=(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)$	M1	
	$= 4 - 2 \times 2 = 0$ (convincing)	A1	
	The equation has 1 real root	B1	
	Any valid reason, eg cubic equations have either 1 or 3 real roots and since $\alpha^2+\beta^2+\gamma^2=0$, not all roots are real	B1	
6(a)	Det(A) = $\lambda(2-\lambda)+2\times4+3(-\lambda-2)$	M1	
	$=-\lambda^2-\lambda+2$	A1	
	A is singular when $-\lambda^2-\lambda+2=0$	M1	
	$\lambda = 1, -2$	A1	
(b)(i)	A = $\begin{bmatrix}-1 & 2 & 3\\-1 & 1 & 1\\2 & -1 & 2\end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix}3 & 4 & -1\\-7 & -8 & 3\\-1 & -2 & 1\end{bmatrix}$ si	M1A1	Award M1 if at least 5 cofactors are correct
	Adjugate matrix = $\begin{bmatrix}3 & -7 & -1\\4 & -8 & -2\\-1 & 3 & 1\end{bmatrix}$	A1	No FT on cofactor matrix
(ii)	Determinant = 2	B1	
	Inverse matrix = $\frac{1}{2}\begin{bmatrix}3 & -7 & -1\\4 & -8 & -2\\-1 & 3 & 1\end{bmatrix}$	B1	FT the adjugate or determinant

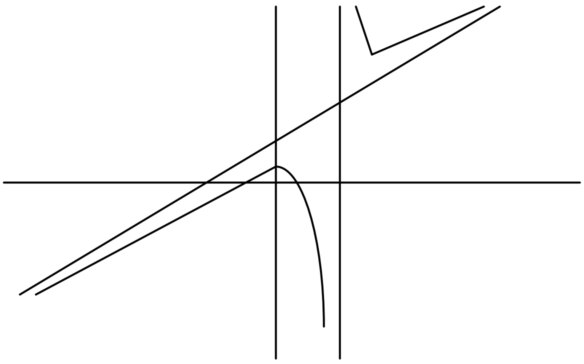
Ques	Solution	Mark	Notes
7(a)	Rotation matrix = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Ref matrix in *y*-axis = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	M1	
	$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	A1	
	$= \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	EITHER		
	The general point on the line is given by $(\lambda, 2\lambda + 1)$	M1	
	Consider $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda \\ 2\lambda+1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2\lambda-2 \\ -\lambda+2 \\ 1 \end{bmatrix}$	m1	
	$x = -2\lambda - 2; y = -\lambda + 2$	A1	
	Eliminating λ, $x - 2y + 6 = 0$ oe	A1	
	OR		
	Consider $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$	M1	
	$-y - 1 = X, -x + 2 = Y$	A1	
	$y = -1 - X, x = 2 - Y$	A1	
	$y = 2x + 1$ leading to $x - 2y + 6 = 0$	A1	

Ques	Solution	Mark	Notes
8	Putting n = 1, the formula gives 1 which is the first term of the series so the result is true for n = 1. Assume formula is true for $n = k$, ie $\left(\sum_{r=1}^{k} r \times 2^{r-1} = 1 + 2^k(k-1) \right)$ Consider, for $n = k + 1$, $\sum_{r=1}^{k+1} r \times 2^{r-1} = \sum_{r=1}^{k} r \times 2^{r-1} + 2^k(k+1)$ $= 1 + 2^k(k-1) + 2^k(k+1)$ $= 1 + 2^{k+1}k$ Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	B1 M1 M1 A1 A1 A1 A1	Award the final A1 only if a correct conclusion is made and the proof is correctly laid out
9(a) (b)	$u + \mathrm{i}v = (x + \mathrm{i}y)(x - 1 + \mathrm{i}y)$ $= x(x-1) - y^2 + \mathrm{i}(xy + xy - y)$ Equating real and imaginary parts, $u = x(x-1) - y^2$ $v = y(2x-1)$ Putting $y = -x$, $u = x(x-1) - x^2 = -x$ $v = -x(2x-1)$ Eliminating x, $v = u(-2u-1)$ cao (oe)	M1 A1 m1 A1 M1 A1 A1 m1 A1	FT their expressions from (a)

Ques	Solution	Mark	Notes
1(a)	$f(-x)=\dfrac{((-x)^2+1)}{-x((-x)^2+2)}=-f(x)$	M1A1	
	Therefore f is odd.	A1	
(b)	Let		
	$\dfrac{x^2+1}{x(x^2+2)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+2}=\dfrac{A(x^2+2)+x(Bx+C)}{x(x^2+2)}$	M1	
	$A=\dfrac{1}{2};B=\dfrac{1}{2};C=0$	A1A1A1	
	$\left(\dfrac{x^2+1}{x(x^2+2)}=\dfrac{1}{2x}+\dfrac{x}{2(x^2+2)}\right)$		
2	$u=\sin^2 x \Rightarrow \mathrm{d}u=2\sin x\cos x\mathrm{d}x$,	B1	
	$[0,\pi/2]\to[0,1]$	B1	
	$I=\int_0^1\dfrac{\mathrm{d}u}{\sqrt{4-u^2}}$	M1	
	$=\left[\sin^{-1}(\dfrac{u}{2})\right]_0^1$	A1	FT a multiple of this
	$=\pi/6$ cao	A1	
3(a)	Denoting the two functional expressions by f_1, f_2		
	$f_1(0)=1,\ f_2(0)=1$	M1A1	
	Therefore f is continuous when $x = 0$.	A1	No FT
(b)	$f_1'(x)=2\mathrm{e}^{2x},\ f_2'(x)=2(1+x)$	M1	
	$f_1'(0)=2,\ f_2'(0)=2$	A1	
	Therefore f' is continuous when $x = 0$.	A1	No FT
4(a)	$\|z\| = 2, \arg(z) = \pi/3$	B1B1	
(b)	Root1 $=\sqrt[3]{2}(\cos\pi/9+\mathrm{i}\sin\pi/9)=1.184+0.431\mathrm{i}$	M1A1	Penalise lack of accuracy once only
	R2 $=\sqrt[3]{2}(\cos 7\pi/9+\mathrm{i}\sin 7\pi/9)=-0.965+0.810\mathrm{i}$	M1A1	
	R3 $=\sqrt[3]{2}(\cos 13\pi/9+\mathrm{i}\sin 13\pi/9)=-0.219-1.241\mathrm{i}$	M1A1	

Ques	Solution	Mark	Notes
5	The equation can be rewritten $2\sin 3\theta \cos 2\theta = \cos 2\theta$ $\cos 2\theta(2\sin 3\theta - 1) = 0$ Either cos2θ = 0, $2\theta = 2n\pi \pm \frac{\pi}{2}$ $\theta = n\pi \pm \frac{\pi}{4}$ Or $\sin 3\theta = 1/2$ $3\theta = n\pi + (-1)^n \frac{\pi}{6}$ or $\theta = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$	**M1A1** **A1** **M1** **A1** **M1** **A1** **A1**	Accept equivalent answers Accept degrees throughout
6	Consider $\cos 6\theta + \mathrm{i}\sin 6\theta = (\cos\theta + \mathrm{i}\sin\theta)^6$ Expanding and equating imaginary terms, isin6θ = $6\cos^5\theta(\mathrm{i}\sin\theta) + 20\cos^3\theta(\mathrm{i}\sin\theta)^3 + 6\cos\theta(\mathrm{i}\sin\theta)^5$ $\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta$ $+ 6\cos\theta\sin^5\theta$ $\frac{\sin 6\theta}{\sin\theta} = 6\cos^5\theta - 20\cos^3\theta(1-\cos^2\theta)$ $+ 6\cos\theta(1-\cos^2\theta)^2$ = $32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$ Letting $\theta \to \pi$ in the right hand side, Limit = – 32 + 32 – 6 = – 6	**M1** **m1** **A1** **A1** **A1** **A1** **M1** **A1**	FT their expression in the line above

Ques	Solution	Mark	Notes
7(a)(i)	The equation can be rewritten as $$\frac{x^2}{9}+\frac{y^2}{4}=1$$	**M1**	
	In the usual notation, a = 3, b = 2.	**A1**	
	$$e=\sqrt{\frac{a^2-b^2}{a^2}}=\frac{\sqrt{5}}{3}$$	**A1**	FT their a,b
(ii)	The foci are $(\pm ae,0)$, ie $(\pm\sqrt{5},0)$ cao	**A1**	
(b)(i)	Substituting the x,y expressions, $$4\times 9\cos^2\theta+9\times 4\sin^2\theta=36(\cos^2\theta+\sin^2\theta)=36$$ showing that P lies on the ellipse.	**B1**	
(ii)	EITHER $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=-\dfrac{2\cos\theta}{3\sin\theta}$ OR $8x+18y\dfrac{dy}{dx}=0;\dfrac{dy}{dx}=-\dfrac{8x}{18y}=-\dfrac{2\cos\theta}{3\sin\theta}$	**M1A1**	
	This equation of the tangent is $$y-2\sin\theta=-\frac{2\cos\theta}{3\sin\theta}(x-3\cos\theta)$$	**M1**	
	$$3y\sin\theta-6\sin^2\theta=-2x\cos\theta+6\cos^2\theta$$ $$3y\sin\theta+2x\cos\theta=6 \quad \text{(convincing)}$$	**A1**	
(iii)	Putting y = 0, R is the point $\left(\dfrac{3}{\cos\theta},0\right)$	**B1**	
	Putting x = 0, S is the point $\left(0,\dfrac{2}{\sin\theta}\right)$	**B1**	
	So M is the point $\left(\dfrac{3}{2\cos\theta},\dfrac{1}{\sin\theta}\right)$	**B1**	
	$$x=\frac{3}{2\cos\theta},\ y=\frac{1}{\sin\theta}$$ Eliminating θ,	**M1**	
	$$\cos\theta=\frac{3}{2x};\sin\theta=\frac{1}{y}$$	**A1**	
	$$\frac{9}{4x^2}+\frac{1}{y^2}=\cos^2\theta+\sin^2\theta=1$$	**A1**	

Ques	Solution	Mark	Notes
8(a)	(0,2) ; (– 4,0) ; (2,0)	**B1**	
(b)(i)	$x = 4$	**B1**	
(ii)	$f(x) = x + 6 + \dfrac{16}{x-4}$	**M1A1**	M1 any valid method
	Oblique asymptote is $y = x + 6$.	**A1**	
(c)	$f'(x) = 1 - \dfrac{16}{(x-4)^2}$ or $\dfrac{x^2-8x}{(x-4)^2}$	**B1**	
	At a stationary point, $f'(x) = 0$	**M1**	
	$(x-4)^2 = 16$ or $x^2 - 8x = 0$	**A1**	
	Stationary points are (0,2) ; (8,18)	**A1**	
(d)		**G1** **G1** **G1**	LH branch RH branch Asymptotes
(e)(i)	$f(-7) = -27/11$; $f(3) = -7$	**M1**	
	$f(S) = [-7,2]$	**A1**	
(ii)	Solve		
	$\dfrac{(x+4)(x-2)}{x-4} = -7$	**M1**	
	$x^2 + 9x - 36 = 0$	**A1**	
	$x = -12, 3$	**A1**	
	$f^{-1}(S) = [-12,3]$	**A1**	

Ques	Solution	Mark	Notes
1(a)	Let $y = \sinh^{-1} x$ so that $x = \sinh y = \dfrac{e^y - e^{-y}}{2}$	**M1**	
	$e^{2y} - 2xe^y - 1 = 0$	**A1**	
	$e^y = \dfrac{2x \pm \sqrt{4x^2 + 4}}{2}$	**A1**	
	$y = \ln\left(x + \sqrt{x^2 + 1}\right)$		
	rejecting the negative sign since $e^y > 0$	**A1**	
(b)	Substituting for cosh2x,		
	$1 + 2\sinh^2 x = 2\sinh x + 5$	**M1**	
	$\sinh^2 x - \sinh x - 2 = 0$	**A1**	
	Solving for sinh$x$,		
	sinh$x$ = –1, 2	**M1A1**	
	$x = \ln(-1 + \sqrt{2}); \ln(2 + \sqrt{5})$	**A1**	
2(a)	Consider		
	$\dfrac{d}{dx}(3 - x)^{1/3} = \dfrac{-(3 - x)^{-2/3}}{3}$	M1A1	
	$= -0.2295\ldots$ when $x = 1.25$	A1	Allow any x between 1.2 and 1.3 M1A0A1 if negative sign omitted
	The sequence converges because this is less than 1 in modulus.	A1	FT the f' value if M1 awarded
	$x_0 = 1.25$		
	$x_1 = 1.205071132$	M1A1	
	$x_2 = 1.215296967$		
	$x_3 = 1.212984693$		
	$x_4 = 1.213508318$		
	$x_5 = 1.21338978$		
	$x_6 = 1.213416617$	A1	
	α = 1.2134 correct to 4 decimal places.	A1	

Ques	Solution	Mark	Notes
(b)	The Newton-Raphson iteration is		
	$x_{n+1} = x_n - \dfrac{(x_n^{\ 3} + x_n - 3)}{3x_n^2 + 1}$ or $\dfrac{2x_n^{\ 3} + 3}{3x_n^{\ 2} + 1}$	M1A1	
	$x_0 = 1.25$		
	$x_1 = 1.214285714$	M1A1	
	$x_2 = 1.213412176$		
	$x_3 = 1.213411663$	A1	
	($x_4 = 1.213411663$)		
	$\alpha = 1.213412$ correct to 6 decimal places	A1	
3(a)	$\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{sech}x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{\cosh x}\right)$		
	$= -\dfrac{\sinh x}{\cosh^2 x} = -\mathrm{sech}x \tanh x$	B1	Convincing
(b)	$f'(x) = \mathrm{sech}^2 x$	B1	
	$f''(x) = -2\mathrm{sech}^2 x \tanh x$	B1	FT 1 slip
	$f'''(x) = 4\mathrm{sech}^2 x \tanh^2 x - 2\mathrm{sech}^4 x$	B1	
	$f(0) = 0,\ f'(0) = 1,\ f''(0) = 0,\ f'''(0) = -2$	B1	
	The Maclaurin series for tanh$x$ is		
	$x - \dfrac{x^3}{3} + \ldots$	M1A1	
(c)	$(1+x)\tanh x \approx x + x^2 - \dfrac{x^3}{3} - \dfrac{x^4}{3}$	B1	FT their series
	$\int_0^{0.5} (1+x)\tanh x \mathrm{d}x \approx \int_0^{0.5} (x + x^2 - \dfrac{x^3}{3} - \dfrac{x^4}{3})\mathrm{d}x$	M1	FT 1 slip
	$= \left[\dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{12} - \dfrac{x^5}{15}\right]_0^{0.5}$	A1	
	= 0.159 cao	A1	

Ques	Solution	Mark	Notes
4	$\mathrm{d}x = \dfrac{2\mathrm{d}t}{1+t^2}$; $[0, \pi/2] \to [0,1]$	B1B1	
	$I = \displaystyle\int_0^1 \dfrac{1}{2-\left(\dfrac{1-t^2}{1+t^2}\right)} \times \dfrac{2\mathrm{d}t}{1+t^2}$	M1A1	
	$= \displaystyle\int_0^1 \dfrac{2}{3t^2+1}\mathrm{d}t$	A1	
	$= \dfrac{2}{3}\displaystyle\int_0^1 \dfrac{1}{t^2+1/3}\mathrm{d}t$	A1	
	$= \dfrac{2\sqrt{3}}{3}\left[\tan^{-1}(t\sqrt{3})\right]_0^1$	A1	
	$= \dfrac{2\sqrt{3}\pi}{9}$ (1.21) cao	A1	
5(a)	$I_n = -\dfrac{1}{2}\displaystyle\int_0^1 x^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{-x^2})\,\mathrm{d}x$	M1	
	$= -\dfrac{1}{2}\left[x^{n-1}\mathrm{e}^{-x^2}\right]_0^1 + \dfrac{n-1}{2}\displaystyle\int_0^1 x^{n-2}\mathrm{e}^{-x^2}\mathrm{d}x$	A1A1	
	$= -\dfrac{\mathrm{e}^{-1}}{2} + \left(\dfrac{n-1}{2}\right)I_{n-2}$		
(b)			
	$I_1 = \displaystyle\int_0^1 x\mathrm{e}^{-x^2}\mathrm{d}x = -\dfrac{1}{2}\left[\mathrm{e}^{-x^2}\right]_0^1$	M1A1	M1A1A1 for evaluating I_1 at any stage
	$= \dfrac{1}{2}\left(1-\mathrm{e}^{-1}\right)$	A1	
	$I_5 = -\dfrac{\mathrm{e}^{-1}}{2} + 2I_3$	M1	
	$= -\dfrac{\mathrm{e}^{-1}}{2} + 2\left(-\dfrac{\mathrm{e}^{-1}}{2} + I_1\right)$	M1	
	$= 1 - 2.5\mathrm{e}^{-1}$	A1	

Ques	Solution	Mark	Notes
6(a)	Consider		
	$y = r\sin\theta$	M1	
	$= (\sin\theta + \cos\theta)\sin\theta$	A1	
	$\frac{dy}{d\theta} = (\cos\theta - \sin\theta)\sin\theta + \cos\theta(\sin\theta + \cos\theta)$	M1	
	$= \sin 2\theta + \cos 2\theta$	A1	FT 1 slip
	The tangent is parallel to the initial line where		
	$\frac{dy}{d\theta} = 0$	M1	
	$\tan 2\theta = -1$	A1	
	$\theta = \frac{3\pi}{8}$ (1.18, 67.5°)	A1	
	$r = 1.31$	A1	
(b)	Area = $\frac{1}{2}\int r^2 d\theta$	M1	
	$= \frac{1}{2}\int_0^{\pi/2}(\sin\theta + \cos\theta)^2 d\theta$	A1	
	$= \frac{1}{2}\int_0^{\pi/2}(1 + \sin 2\theta)d\theta$	A1	
	$= \frac{1}{2}\left[\theta - \frac{1}{2}\cos 2\theta\right]_0^{\pi/2}$	A1	
	$= \frac{\pi}{4} + \frac{1}{2}$ (1.29) cao	A1	

Ques	Solution	Mark	Notes
7(a)	$x = a\sinh\theta \to \mathrm{d}x = a\cosh\theta\mathrm{d}\theta$	B1	
	$I = \int\sqrt{a^2(1+\sinh^2\theta)}\; a\cosh\theta\mathrm{d}\theta$	M1	
	$= a^2\int\cosh^2\theta\,\mathrm{d}\theta$	A1	
	$= \dfrac{a^2}{2}\int(1+\cosh 2\theta)\,\mathrm{d}\theta$	A1	
	$= \dfrac{a^2}{2}\left(\theta+\sinh\theta\cosh\theta\right)$	A1	FT line above
	$= \dfrac{a^2}{2}\left(\sinh^{-1}\left(\dfrac{x}{a}\right)+\dfrac{x\sqrt{x^2+a^2}}{a^2}\right)$ (+ C)		Answer given
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x$	B1	
	$L = \int\sqrt{1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\mathrm{d}x$	M1	
	$= \int_0^1\sqrt{1+4x^2}\;\mathrm{d}x$	A1	
	$= 2\int_0^1\sqrt{\left(x^2+1/4\right)}\mathrm{d}x$	A1	
	$= \dfrac{2}{8}\left[\sinh^{-1}2x+4x\sqrt{x^2+1/4}\right]_0^1$	A1	
	$= 1.48$	A1	

WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk