GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2013

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2013 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

1. (*a*) (i) Gradient of $BC = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = -4$ (or equivalent) A1

(ii) A correct method for finding the equation of BC using candidate's gradient for BC M1

Equation of BC : $y - (-5) = -4(x - 6)$ (or equivalent)
(f.t. candidate's gradient of BC) A1

Equation of BC : $4x + y - 19 = 0$ (convincing) A1

(iii) Use of $m_{AD} \times m_{BC} = -1$ M1

A correct method for finding the equation of AD using candidate's gradient for AD (M1)

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of AD : $y - 4 = {}^{1}/_{4}(x - 8)$ (or equivalent)
(f.t. candidate's gradient of BC) A1

Note: Total mark for part (*a*) is 7 marks

(*b*) An attempt to solve equations of BC and AD simultaneously M1

$x = 4$, $y = 3$ (convincing) (c.a.o.) A1

(*c*) A correct method for finding the length of BD M1

$BD = \sqrt{68}$ A1

(*d*) A correct method for finding E M1

$E(0, 2)$ A1

2. (*a*) $\dfrac{2 + 5\sqrt{7}}{4 + \sqrt{7}} = \dfrac{(2 + 5\sqrt{7})(4 - \sqrt{7})}{(4 + \sqrt{7})(4 - \sqrt{7})}$ M1

Numerator: $8 - 2\sqrt{7} + 20\sqrt{7} - 35$ A1

Denominator: $16 - 7$ A1

$\dfrac{2 + 5\sqrt{7}}{4 + \sqrt{7}} = -3 + 2\sqrt{7}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 + \sqrt{7}$

(*b*) $\sqrt{360} = 6\sqrt{10}$ B1

$\sqrt{2} \times (\sqrt{5})^3 = 5\sqrt{10}$ B1

$\dfrac{\sqrt{30} \times \sqrt{8}}{\sqrt{6}} = 2\sqrt{10}$ B1

$\sqrt{360} - \sqrt{2} \times (\sqrt{5})^3 - \dfrac{\sqrt{30} \times \sqrt{8}}{\sqrt{6}} = -\sqrt{10}$ (c.a.o.) B1

3. (*a*) $\frac{dy}{dx} = 4x - 10$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 2$ (c.a.o.) A1

Use of gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal at P: $y - (-5) = -\,{}^{1}/_{2}(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1

(*b*) An attempt to put candidate's expression for $\frac{dy}{dx} = 0$ M1

x-coordinate of $Q = 2{\cdot}5$

(f.t. one error in candidate's expression for $\frac{dy}{dx}$) A1

4. (*a*) $2(x - 4)^2 - 40$ B1 B1 B1

(*b*) least value $= -20$ (f.t. candidate's value for c) B1

x-coordinate $= 4$ (f.t. candidate's value for b) B1

5. (*a*) $(1 + 2x)^7 = 1 + 14x + 84x^2 \ldots$ B1 B1 B1

(*b*) $(1 - 4x)(1 + 2x)^7 = 1 - 4x + 14x - 56x^2 + 84x^2$

Constant term and terms in x B1

Terms in x^2 B1

(f.t. candidate's expression in (*a*))

$(1 - 4x)(1 + 2x)^7 = 1 + 10x + 28x^2$ (c.a.o.) B1

6. (*a*) (i) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = (4k + 1)^2 - 4 \times (k + 1) \times (k - 5)$ A1

Putting $b^2 - 4ac = 0$ m1

$4k^2 + 8k + 7 = 0$ (convincing) A1

(ii) An expression for $b^2 - 4ac$, with at least two of a, b, c correct (M1)

(to be awarded only if corresponding M1 is not awarded in part (i))

$b^2 - 4ac = 64 - 112\ (= -48)$ A1

$b^2 - 4ac < 0 \Rightarrow$ no real roots A1

Note: Total mark for part (*a*) is 6 marks

(*b*) Finding critical values $x = -{}^3/_4$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that

$x \le -{}^3/_4$ or $3 \le x$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 5(x + \delta x)^2 + 8(x + \delta x) - 11$ B1

Subtracting y from above to find δy M1

$\delta y = 10x\delta x + 5(\delta x)^2 + 8\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 8$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 6 \times \frac{2}{3} \times x^{-1/3} + 5 \times -2 \times x^{-3}$ (completely correct answer) B2

(**If B2 not awarded**, award B1 for at least one correct non-zero term)

8. Attempting to find $f(r) = 0$ for some value of r M1

$f(-1) = 0 \Rightarrow x + 1$ is a factor A1

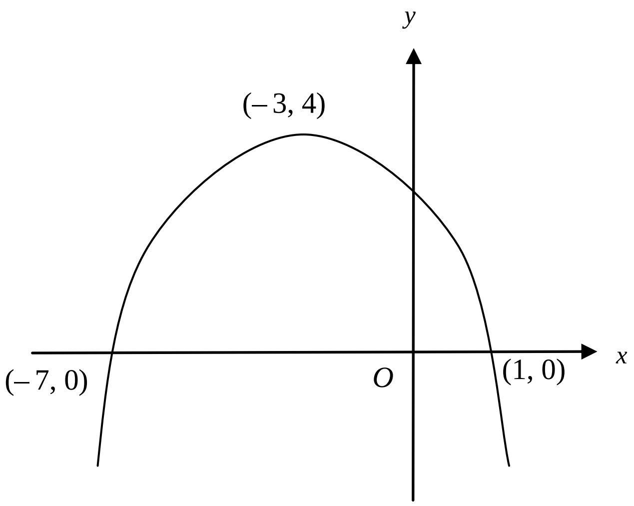
$f(x) = (x + 1)(8x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 1)(8x^2 - 10x + 3)$ A1

$f(x) = (x + 1)(2x - 1)(4x - 3)$ (f.t. only $8x^2 + 10x + 3$ in above line) A1

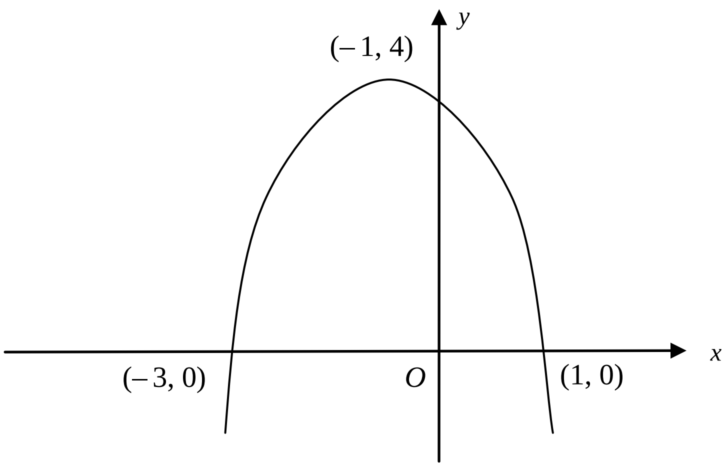
$x = -1, {}^1/_2, {}^3/_4$ (f.t. for factors $2x \pm 1$, $4x \pm 3$) A1

9. (*a*)



Concave down curve with *y*-coordinate of maximum = 4	B1
x-coordinate of maximum = – 3	B1
Both points of intersection with *x*-axis	B1

(*b*)



Concave down curve with *y*-coordinate of maximum = 4	B1
x-coordinate of maximum = – 1	B1
Both points of intersection with *x*-axis	B1

Note: A candidate who draws a curve with no changes to the original graph is awarded 0 marks (both parts)

10. (*a*) (i) $(2x \times x) + (2x \times x) + (2x \times y) + (2x \times y) + (x \times y) + (x \times y)$
$= 108$ M1

$6xy + 4x^2 = 108 \Rightarrow xy = 18 - \frac{2x^2}{3}$ (convincing) A1

(ii) $V = 2x \times x \times y = 2x(xy) \Rightarrow V = 36x - \frac{4x^3}{3}$ (convincing) B1

(*b*) $\frac{dV}{dx} = 36 - 3 \times \frac{4x^2}{3}$ B1

Putting derived $\frac{dV}{dx} = 0$ M1

$x = 3, (-3)$ (f.t. candidate's $\frac{dV}{dx}$) A1

Stationary value of V at $x = 3$ is 72 (c.a.o) A1

A correct method for finding nature of the stationary point yielding a maximum value (for $0 < x$) B1

C2

1.

0	0·5		
0·5	0·470588235		
1	0·333333333		
1·5	0·186046511		
2	0·1	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ — M1

$I \approx \frac{0·5}{2} \times \{0·5 + 0·1 + 2(0·470588235 + 0·333333333 + 0·186046511)\}$

$I \approx 2·579936152 \times 0·5 \div 2$

$I \approx 0·644984038$

$I \approx 0·645$ (f.t. one slip) — A1

Special case for candidates who put $h = 0·4$

0	0·5		
0·4	0·484496124		
0·8	0·398089172		
1·2	0·268240343		
1·6	0·164041994		
2	0·1	(all values correct)	B1

Correct formula with $h = 0·4$ — M1

$I \approx \frac{0·4}{2} \times \{0·5 + 0·1 + 2(0·484496124 + 0·398089172 + 0·268240343 + 0·164041994)\}$

$I \approx 3·229735266 \times 0·4 \div 2$

$I \approx 0·645947053$

$I \approx 0·646$ (f.t. one slip) — A1

Note: Answer only with no working earns 0 marks

2. (*a*) (i) Correct use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (o.e.) — M1

Correct use of $\cos^2\theta = 1 - \sin^2\theta$ — M1

$6(1 - \sin^2\theta) + 5\sin\theta = 0 \Rightarrow 6\sin^2\theta - 5\sin\theta - 6 = 0$

(convincing) — A1

(ii) An attempt to solve given quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c = 6$ and $b \times d = -6$ — M1

$6\sin^2\theta - 5\sin\theta - 6 = 0 \Rightarrow (3\sin\theta + 2)(2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{2}{3}$, $(\sin\theta = {}^3/_2)$ (c.a.o.) — A1

$\theta = 221·81°, 318·19°$ — B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $3\sin\theta + 2 = 0$, ignore roots outside range.

$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

(*b*) $2x - 60° = -38°, 38°, 322°$ (one value) — B1

$x = 11°, 49°$ — B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) Either: $(x + 2)^2 = x^2 + (x - 2)^2 - 2 \times x \times (x - 2) \times \cos B\hat{A}C$

Or: $\cos B\hat{A}C = \dfrac{x^2 + (x - 2)^2 - (x + 2)^2}{2 \times x \times (x - 2)}$

(substituting the correct expressions in the correct places in the cos rule) M1

Either: $\cos B\hat{A}C = \dfrac{x^2 + x^2 - 4x + 4 - x^2 - 4x - 4}{2 \times x \times (x - 2)}$ (o.e.)

Or: $\cos B\hat{A}C = \dfrac{x^2 + x^2 - 4x + 4 - x^2 - 4x - 4}{2x^2 - 4x}$ (o.e.) A1

$\cos B\hat{A}C = \dfrac{x - 8}{2x - 4}$ (convincing) A1

(*b*) (i) $\dfrac{x - 8}{2x - 4} = -\dfrac{1}{2}$ M1

$x = 5$ A1

(ii) **Either:**

$\dfrac{\sin ABC}{3} = \dfrac{\sin 120°}{7}$

(substituting the correct values in the correct places in the sin rule, f.t. candidate's value for x, provided $x > 2$) M1

$ABC = 21{\cdot}8°$

(f.t. candidate's value for x, provided $x > 2$) A1

Or:

$3^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos ABC$

(substituting the correct values in the correct places in the cos rule, f.t. candidate's value for x, provided $x > 2$) M1

$ABC = 21{\cdot}8°$

(f.t. candidate's value for x, provided $x > 2$) A1

4. (*a*) $S_n = a + [a + d] + . . . + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + . . . + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + . . . + [a + a + (n - 1)d]$

(at least three terms, including those derived from the first pair and the last pair plus one other pair of terms)

Or:

$2S_n = [a + a + (n - 1)d] + . . .$ (n times) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \dfrac{n}{2}[2a + (n - 1)d]$ (convincing) A1 (*b*) **Either:**

$\frac{10}{2}(2a + 9d) = 115$ B1

$S_{14} = 115 + 130$ M1

$\frac{14}{2}(2a + 13d) = 245$ A1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -2$, $d = 3$ (both values) (c.a.o.) A1

Or:

$\frac{10}{2}(2a + 9d) = 115$ B1

$(a + 10d) + (a + 11d) + (a + 12d) + (a + 13d) = 130$ M1

$4a + 46d = 130$ (seen or implied by later work) A1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -2$, $d = 3$ (both values) (c.a.o.) A1

5. (*a*) $r = 0{\cdot}8$ B1

$S_{18} = \frac{100(1 - 0{\cdot}8^{18})}{1 - 0{\cdot}8}$ M1

$S_{18} \approx 490{\cdot}992 = 491$ (c.a.o.) A1

(*b*) (i) $ar = -20$ B1

$\frac{a}{1 - r} = 64$ B1

An attempt to solve these equations simultaneously by eliminating *a* M1

$16r^2 - 16r - 5 = 0$ (convincing) A1

(ii) $r = -{}^1/_4$ (c.a.o.) B1

$|r| < 1$ E1

6. (*a*) $\dfrac{x^{5/4}}{5/4} + 2 \times \dfrac{x^{-4}}{-4} + c$ B1,B1

(– 1 if no constant present)

(*b*) (i) $x^2 + 3 = 4x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 3$ m1

$(x - 1)(x - 3) = 0 \Rightarrow x = 1, x = 3$ (both values, c.a.o) A1

Note: Answer only with no working earns 0 marks

(ii) Area of small triangle = 2

(any method, f.t. candidate's value for x_A) B1

Use of integration to find the area under the curve M1

$\int x^2\,dx = (1/3)x^3$, $\int 3\,dx = 3x$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$$\left[(1/3)x^3 + 3x\right]_1^3 = (9 + 9) - (1/3 + 3) = 44/3$$

Use of candidate's values for x_A and x_B as limits and trying to find total area by adding area under curve to area of triangle

m1

Shaded area = 44/3 + 2 = 50/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indices) B1

$\log_a xy = p + q$ (the relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) **Either:**

$(2 - 3x) \log_{10} 5 = \log_{10} 8$

(taking logs on both sides and using the power law) M1

$x = \dfrac{2 \log_{10} 5 - \log_{10} 8}{3 \log_{10} 5}$ A1

$x = 0{\cdot}236$ (f.t. one slip, see below) A1

Or:

$2 - 3x = \log_5 8$ (rewriting as a log equation) M1

$x = \dfrac{2 - \log_5 8}{3}$ A1

$x = 0{\cdot}236$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}236$ from $x = \dfrac{\log_{10} 8 - 2 \log_{10} 5}{3 \log_{10} 5}$

earns M1 A0 A1

an answer of $x = 1{\cdot}097$ from $x = \dfrac{2 \log_{10} 5 + \log_{10} 8}{3 \log_{10} 5}$

earns M1 A0 A1

an answer of $x = 0{\cdot}708$ from $x = \dfrac{2 \log_{10} 5 - \log_{10} 8}{\log_{10} 5}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) $\frac{1}{2}\log_a 144x^8 = \log_a 12x^4$ (power law) B1

$\log_a 90x^2 - \log_a\left(\frac{5}{x}\right) = \log_a\left(\frac{90x^2 \times x}{5}\right)$ (subtraction law) B1

$\frac{90x^2 \times x}{5} = 12x^4$ (removing logs, f.t one incorrect term) B1

$x = 1\cdot5$ (c.a.o.) B1

8. (*a*) $A(-1, 3)$ B1

A correct method for finding the radius M1

Radius = 5 A1

(*b*) (i) Showing that the coordinates of A do not satisfy the equation of L (f.t. candidate's coordinates for A) B1

(ii) An attempt to substitute $(9 - x)$ for y in the equation of C_1 M1

$x^2 - 5x + 6 = 0$ (or $2x^2 - 10x + 12 = 0$) A1

$x = 2, x = 3$

(correctly solving candidate's quadratic, both values) A1

Points of intersection are (2, 7), (3, 6) (c.a.o.) A1

(*c*) Distance between centres of C_1 and $C_2 = 13$

(f.t. candidate's coordinates for A) B1

Use of the fact that the shortest distance between the circles = distance between centres – sum of the radii M1

Shortest distance between the circles = 2

(f.t. candidate's coordinates for A and radius for C_1.) A1

9. (*a*) Substitution of values in area formula for triangle M1

Area = $\frac{1}{2} \times 7\cdot2^2 \times \sin 1\cdot1 = 23\cdot1\ \text{cm}^2$. A1

(*b*) Let $B\hat{O}C = \phi$ radians

$\frac{1}{2} \times 7\cdot2^2 \times \phi = 19\cdot44$ M1

$\phi = 0\cdot75$ (o.e.) A1

Length of arc $BC = 7\cdot2 \times 0\cdot75 = 5\cdot4$ cm

(f.t. candidate's value for ϕ) A1

1. (*a*)

1	1·945910149
1·5	2·238046572
2	2·63905733
2·5	3·073850053
3	3·496507561

(5 values correct) B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{3} \times \{1·945910149 + 3·496507561 + 4(2·238046572 + 3·073850053) + 2(2·63905733)\}$

$I \approx 31·96811887 \times 0·5 \div 3$

$I \approx 5·328019812$

$I \approx 5·328$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\int_1^3 \ln \sqrt{(x^3 + 6)}\, dx \approx 2·664$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $4(\text{cosec}^2\theta - 1) - 8 = 2\,\text{cosec}^2\theta - 5\,\text{cosec}\,\theta$

(correct use of $\cot^2\theta = \text{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\,\text{cosec}\,\theta + b)(c\,\text{cosec}\,\theta + d)$, with $a \times c$ = coefficient of $\text{cosec}^2\theta$ and $b \times d$ = candidate's constant

m1

$2\,\text{cosec}^2\theta + 5\,\text{cosec}\,\theta - 12 = 0 \Rightarrow (2\,\text{cosec}\,\theta - 3)(\text{cosec}\,\theta + 4) = 0$

$\Rightarrow \text{cosec}\,\theta = \frac{3}{2}, \text{cosec}\,\theta = -4$

$\Rightarrow \sin\theta = \frac{2}{3}, \sin\theta = -\frac{1}{4}$ (c.a.o.) A1

$\theta = 41·81°, 138·19°$ B1

$\theta = 194·48°, 345·52°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

(*b*) Correct use of $\sec\phi = \frac{1}{\cos\phi}$ and $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\sin\phi = -\frac{1}{2}$ A1

$\phi = 210°, 330°$ (f.t. for $\sin\phi = -a$) A1

3. (*a*) Use of product formula yielding $x^3 \times 2y \times \frac{dy}{dx} + 3x^2 \times y^2$ B1 B1

$\frac{dy}{dx} = -\frac{3x^2y^2}{2x^3y}$ (c.a.o.) B1

(*b*) (i) Putting candidate's expression for $\frac{dy}{dx} = 3$ and an attempt to simplify M1

$-\frac{3a^2b^2}{2a^3b} = 3 \Rightarrow b = -2a$ (convincing) A1

(ii) Substituting a for x and $-2a$ for y in the equation for C M1

$a = 2$, $b = -4$ A1

4. (*a*) Differentiating ln t and $5t^4$ with respect to t, at least one correct M1

candidate's x-derivative = $\frac{1}{t}$,

candidate's y-derivative = $20t^3$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 20t^4$ (c.a.o.) A1

(*b*) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 80t^3$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = 80t^4$ (f.t. one slip) A1

$\frac{d^2y}{dx^2} = 0{\cdot}648 \Rightarrow t = 0{\cdot}3$ (c.a.o.) A1

5. (*a*) $\frac{dy}{dx} = 5 \times (7 - 9x^2)^4 \times f(x)$, $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = -90x \times (7 - 9x^2)^4$ A1

(*b*) $\frac{dy}{dx} = \boldsymbol{\frac{6}{1 + (6x)^2}}$ **or** $\frac{1}{1 + (6x)^2}$ **or** $\frac{6}{1 + 6x^2}$ M1

$\frac{dy}{dx} = \frac{6}{1 + 36x^2}$ A1

(*c*) $\frac{dy}{dx} = e^{4x} \times m \sec^2 2x + \tan 2x \times ke^{4x}$ ($m = 1$, **2**, $k = 1$, **4**) M1

$\frac{dy}{dx} = e^{4x} \times 2 \sec^2 2x + \tan 2x \times 4e^{4x}$ (at least one correct term) B1

$\frac{dy}{dx} = e^{4x} \times 2 \sec^2 2x + \tan 2x \times 4e^{4x}$ (c.a.o.) A1 (*d*) $\frac{dy}{dx} = \frac{(2 + \cos x) \times m \cos x - (3 + \sin x) \times k \sin x}{(2 + \cos x)^2}$ ($m = \mathbf{1}, -1$ $k = 1, \mathbf{-1}$) M1

$\frac{dy}{dx} = \frac{(2 + \cos x) \times (\cos x) - (3 + \sin x) \times (-\sin x)}{(2 + \cos x)^2}$ A1

$\frac{dy}{dx} = \frac{2\cos x + 3\sin x + 1}{(2 + \cos x)^2}$ A1

6. (*a*) (i) $\int \cos(3x + \pi/2)\,dx = k \times \sin(3x + \pi/2) + c$

($k = 1, 3, \mathbf{{}^1/_3}, -{}^1/_3$) M1

$\int \cos(3x + \pi/2)\,dx = {}^1/_3 \times \sin(3x + \pi/2) + c$ A1

(ii) $\int e^{3-4x}\,dx = k \times e^{3-4x} + c$ ($k = 1, -4, {}^1/_4, \mathbf{-{}^1/_4}$) M1

$\int e^{3-4x}\,dx = -{}^1/_4 \times e^{3-4x} + c$ A1

(iii) $\int \frac{7}{8x + 5}\,dx = 7 \times k \times \ln|8x + 5| + c$ ($k = 1, 8, \mathbf{{}^1/_8}$) M1

$\int \frac{7}{8x + 5}\,dx = 7 \times {}^1/_8 \times \ln|8x + 5| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (2x - 1)^{-4}\,dx = k \times \frac{(2x - 1)^{-3}}{-3}$ ($k = 1, 2, \mathbf{{}^1/_2}$) M1

$\int_1^2 9 \times (2x - 1)^{-4}\,dx = \left[9 \times {}^1/_2 \times \frac{(2x - 1)^{-3}}{-3}\right]_1^2$ A1

Correct method for substitution of limits in an expression of the form $m \times (2x - 1)^{-3}$ M1

$\int_1^2 9 \times (2x - 1)^{-4}\,dx = \frac{13}{9} = 1{\cdot}44$ (f.t. for $k = 1, 2$ only) A1

Note: Answer only with no working earns 0 marks

7.	(a)	Choice of $a \neq -1$ and $b = -a - 2$		M1
		Correct verification that given equation is satisfied		A1
	(b)	Trying to solve either $x^2 - 10 \leq 6$ or $x^2 - 10 \geq -6$		M1
		$x^2 - 10 \leq 6 \Rightarrow x^2 \leq 16$		
		$x^2 - 10 \geq -6 \Rightarrow x^2 \geq 4$	(both inequalities)	A1
		At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$	(f.t. one slip)	A1
		Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$	(c.a.o.)	A1
		Alternative mark scheme		
		$(x^2 - 10)^2 \leq 36$	(forming and trying to solve quadratic in x^2)	M1
		Critical values $x^2 = 4$ and $x^2 = 16$		A1
		At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$	(f.t. one slip)	A1
		Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$	(c.a.o.)	A1

8. $x_0 = -1{\cdot}5$

$x_1 = -1{\cdot}666394263$ (x_1 correct, at least 5 places after the point) B1

$x_2 = -1{\cdot}676625462$

$x_3 = -1{\cdot}677198866$

$x_4 = -1{\cdot}677230823 = -1{\cdot}67723$ (x_4 correct to 5 decimal places) B1

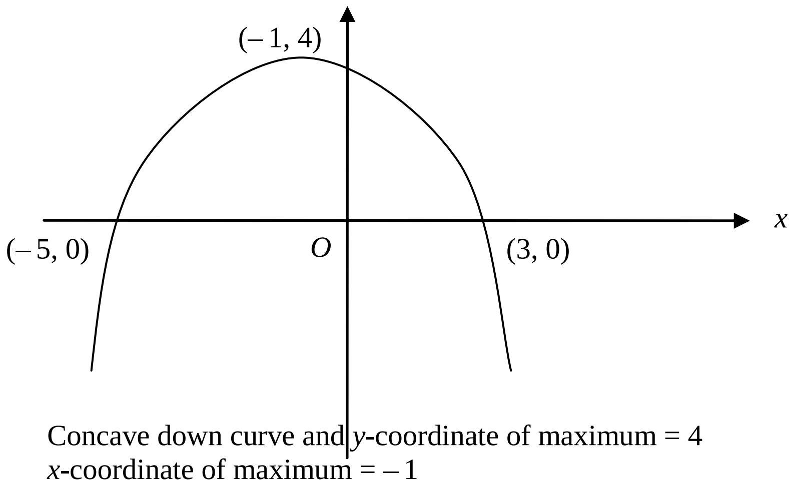
Let $f(x) = x^2 + e^x - 3$

An attempt to check values or signs of $f(x)$ at $x = -1{\cdot}677225$, $x = -1{\cdot}677235$ M1

$f(-1{\cdot}677225) = -2{\cdot}44 \times 10^{-5} < 0$, $f(-1{\cdot}677235) = 7{\cdot}26 \times 10^{-6} > 0$ A1

Change of sign $\Rightarrow \alpha = -1{\cdot}67723$ correct to five decimal places A1

9.



Concave down curve and y-coordinate of maximum = 4 B1

x-coordinate of maximum = – 1 B1

Both points of intersection with x-axis B1

10.	(*a*)	$y - 6 = e^{5 - x/2}$.		B1
		An attempt to express equation as a logarithmic equation and to isolate x		M1
		$x = 2\,[5 - \ln(y - 6)]$	(c.a.o.)	A1
		$f^{-1}(x) = 2\,[5 - \ln(x - 6)]$	(f.t. one slip in candidate's expression for x)	A1
	(*b*)	$D(f^{-1}) = [7, \infty)$		B1 B1

11.	(*a*)	(i)	$D(fg) = (0, \pi/4]$	B1
		(ii)	$R(fg) = (-\infty, 0]$	B1 B1
	(*b*)	(i)	$fg(x) = -0{\cdot}4 \Rightarrow \tan x = e^{-0{\cdot}4}$	M1
			$x = 0{\cdot}59$	A1
		(ii)	Equation has solution only if $k \in R(fg)$.	
			$\therefore$ choose any $k \notin R(fg)$ (f.t. candidate's $R(fg)$)	B1

1. (*a*) $f(x) \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x+2)}$ (correct form) M1

$6 + x - 9x^2 \equiv A(x+2) + Bx(x+2) + Cx^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3$, $C = -8$, $B = -1$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) (i) $f'(x) = \frac{-6}{x^3} + \frac{1}{x^2} + \frac{8}{(x+2)^2}$ (o.e.)

(f.t. candidate's values for *A*, *B*, *C*)

(first term) B1

(at least one of last two terms) B1

(ii) $f'(2) = 0 \Rightarrow$ stationary value when $x = 2$ (c.a.o.) B1

2. $3x^2 - 2x \times 2y\frac{dy}{dx} - 2y^2 + 3y^2\frac{dy}{dx} = 0$ $\left[-2x \times 2y\frac{dy}{dx} - 2y^2\right]$ B1

$\left[3x^2, 3y^2\frac{dy}{dx}\right]$ B1

Either $\frac{dy}{dx} = \frac{2y^2 - 3x^2}{3y^2 - 4xy}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 1 = -\frac{1}{2}(x - 2)$ $\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ A1

3. (*a*) $8(2\cos^2\theta - 1) + 6 = \cos^2\theta + \cos\theta$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (5\cos\theta - 2)(3\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{5}$, $\cos\theta = -\frac{1}{3}$, (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks

$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $R = 4$ B1

Correctly expanding $\cos(\theta + \alpha)$, correctly comparing coefficients and using either $4\cos\alpha = \sqrt{15}$ **or** $4\sin\alpha = 1$ **or** $\tan\alpha = \dfrac{1}{\sqrt{15}}$ to find α

(f.t. candidate's value for *R*) M1

$\alpha = 14{\cdot}48°$ (c.a.o.) A1

$\cos(\theta + 14{\cdot}48°) = \dfrac{3}{4} = 0{\cdot}75$

(f.t. candidate's values for *R*, α, $0° < \alpha < 90°$) B1

$\theta + 14{\cdot}48° = 41{\cdot}41°, 318{\cdot}59°$

(at least one value, f.t. candidate's values for *R*, α, $0° < \alpha < 90°$) B1

$\theta = 26{\cdot}93°, 304{\cdot}11°$ (c.a.o.) B1

4.

$\text{Volume} = \pi\displaystyle\int_{\pi/6}^{\pi/2} \sin^2 2x \, dx$ B1

$\sin^2 2x = \dfrac{(1 - \cos 4x)}{2}$ B1

$\displaystyle\int (a + b\cos 4x)\, dx = ax + ¼\, b\sin 4x$, $a \neq 0, b \neq 0$ B1

Correct substitution of candidate's limits in candidate's integrated expression of form $mx + n\sin 4x$ $m \neq 0, n \neq 0$ M1

Volume $= 1{\cdot}985$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. (*a*) (i) $(1 + 6x)^{1/3} = 1 + 2x - 4x^2$ $(1 + 2x)$ B1

$(-4x^2)$ B1

(ii) $|x| < {}^1/_6$ or $-{}^1/_6 < x < {}^1/_6$ B1

(*b*) $2 + 4x - 8x^2 = 2x^2 - 15x \Rightarrow 10x^2 - 19x - 2 = 0$ M1

(An attempt to set up and use a correct method to solve quadratic using candidate's expansion for $(1 + 6x)^{1/3}$)

$x = -0{\cdot}1$ (f.t. only candidate's range for *x* in (*a*)) A1

6. (*a*) candidate's x-derivative = a

candidate's y-derivative = $-\dfrac{b}{t^2}$ (at least one term correct) B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = -\dfrac{b}{at^2}$ (c.a.o.) A1

Tangent at P: $y - \dfrac{b}{p} = -\dfrac{b}{ap^2}(x - ap)$ (o.e.)

(f.t. candidate's expression for $\dfrac{dy}{dx}$) M1

$ap^2y - abp = -bx + abp$

$ap^2y + bx - 2abp = 0.$ (convincing) A1

(*b*) $y = 0 \Rightarrow x = 2ap$ (o.e.) B1

$x = 0 \Rightarrow y = 2b/p$ (o.e.) B1

Area of triangle $AOB = 2ab$ (c.a.o.) B1

(*c*) $p^2 - 2p + 2 = 0$ $(abp^2 - 2abp + 2ab = 0)$ B1

Attempting **either** to use the formula to solve the candidate's quadratic in p **or** to find the discriminant of the candidate's quadratic **or** to complete the square M1

Either discriminant < 0 ($\Rightarrow$ no real roots) $\Rightarrow$ no such P can exist

or $(p-1)^2 + 1 = 0$ $(\Rightarrow (p-1)^2 = -1)) \Rightarrow$ no such P can exist

(c.a.o.) A1

7. (*a*) $u = 3x - 1 \Rightarrow du = 3dx$ (o.e.) B1

$dv = \cos 2x\, dx \Rightarrow v = \dfrac{1}{2}\sin 2x$ (o.e.) B1

$\int(3x-1)\cos 2x\, dx = (3x-1) \times \dfrac{1}{2}\sin 2x - \int \dfrac{1}{2}\sin 2x \times 3dx$ M1

$\int(3x-1)\cos 2x\, dx = \dfrac{1}{2}(3x-1)\sin 2x + \dfrac{3}{4}\cos 2x + c$ (c.a.o.) A1

(*b*) $\int \dfrac{x}{(2x+1)^3}dx = \int \dfrac{f(u)}{u^3} \times k\, du$

($f(u) = pu + q$, $p \neq 0$, $q \neq 0$ and $k = \tfrac{1}{2}$ or 2) M1

$\int \dfrac{x}{(2x+1)^3}dx = \int \dfrac{(u-1)}{2} \times \dfrac{1}{u^3} \times \dfrac{du}{2}$ A1

$\int (au^{-2} + bu^{-3})\, du = \dfrac{au^{-1}}{-1} + \dfrac{bu^{-2}}{-2}$ ($a \neq 0$, $b \neq 0$) B1

Either: Correctly inserting limits of 1, 3 in candidate's $cu^{-1} + du^{-2}$

($c \neq 0$, $d \neq 0$)

or: Correctly inserting limits of 0, 1 in candidate's

$c(2x+1)^{-1} + d(2x+1)^{-2}$ ($c \neq 0$, $d \neq 0$) m1

$\int_0^1 \dfrac{x}{(2x+1)^3}dx = \dfrac{1}{18}$ (= 0·055 . . .) (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $\frac{dA}{dt} = k\sqrt{A}$ B1

(*b*) $\int \frac{dA}{\sqrt{A}} = \int k\,dt$ M1

$\frac{A^{1/2}}{1/2} = kt + c$ A1

Substituting 64 for A and 3 for t and 196 for A and 5·5 for t in candidate's derived equation m1

$16 = 3k + c$, $28 = 5{\cdot}5k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c m1

$A = (2{\cdot}4t + 0{\cdot}8)^2$ (o.e.) (c.a.o.) A1

9. (*a*) $\mathbf{AB} = 8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$ B1

(*b*) $\mathbf{OC} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + {}^3/_4(8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$ (o.e.) M1

$\mathbf{OC} = 5\mathbf{i} + 2\mathbf{k}$ A1

(*c*) (i) Use of $\mathbf{OA} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ on r.h.s. M1

$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ (all correct) A1

(ii) $-1 + \lambda \times (-4) = 7$

(an equation in λ from one set of coefficients) M1

$\lambda = -2$ A1

$1 + (-2) \times 1 = -1$

$11 + (-2) \times 3 = 5$ (both verifications) A1

An attempt to evaluate $\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ M1

$\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 0$ (convincing) A1

B lies on L, AB is perpendicular to L and thus B is the foot of the perpendicular from A to L (c.a.o.) A1

10. Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$

$25x^2 - 30x + 9 + 1 < 9x^2 - 6x + 1 \Rightarrow 16x^2 - 24x + 9 < 0$ B1

$(4x - 3)^2 < 0$ B1

This contradicts the fact that x is real and thus $(5x - 3)^2 + 1 \geq (3x - 1)^2$. B1

Ques	Solution	Mark	Notes
1	$S_n = \sum_{r=1}^{n}(2r-1)^2 = \sum_{r=1}^{n}4r^2 - \sum_{r=1}^{n}4r + \sum_{r=1}^{n}1$	**M1A1**	M1A0 for 2 correct terms
	$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$	**A1A1**	Award A1 for 2 correct
	$= \frac{n}{6}\left(8n^2 + 12n + 4 - 12n - 12 + 6\right)$	**A1**	FT line above if at least 2 terms present
	$= \frac{4n^3}{3} - \frac{n}{3}$ cao	**A1**	Penalise 1 mark if n used as dummy variable in summations
2(a)	EITHER $\frac{1}{w} = \frac{1}{1-\mathrm{i}} + \frac{1}{1+2\mathrm{i}}$		
	$= \frac{1+2\mathrm{i}+1-\mathrm{i}}{(1-\mathrm{i})(1+2\mathrm{i})}$	**M1A1**	
	$= \frac{2+\mathrm{i}}{3+\mathrm{i}}$	**A1**	
	$w = \frac{3+\mathrm{i}}{2+\mathrm{i}} \times \frac{2-\mathrm{i}}{2-\mathrm{i}}$	**M1**	
	$= \frac{7-\mathrm{i}}{5}$	**A1A1**	1 each for num and denom
	OR $\frac{1}{1-\mathrm{i}} = \frac{1+\mathrm{i}}{1-\mathrm{i}^2} = \frac{1+\mathrm{i}}{2}$	**M1A1**	
	$\frac{1}{1+2\mathrm{i}} = \frac{1-2\mathrm{i}}{1-4\mathrm{i}^2} = \frac{1-2\mathrm{i}}{5}$	**A1**	
	$\frac{1}{w} = \frac{5+5\mathrm{i}+2-4\mathrm{i}}{10} = \frac{7+\mathrm{i}}{10}$	**A1**	
	$w = \frac{10}{7+\mathrm{i}} \times \frac{7-\mathrm{i}}{7-\mathrm{i}}$	**M1**	
	$= \frac{7-\mathrm{i}}{5}$	**A1**	1 each for num and denom
(b)	$\text{Mod}(w) = \frac{\sqrt{50}}{5}$ $(\sqrt{2})$	**B1**	FT on their w
	$\text{Arg}(w) = -0.142$ $(-8.13°)$	**B1**	Accept 351.9° or 6.14 Do not FT arg if in 1st quadrant

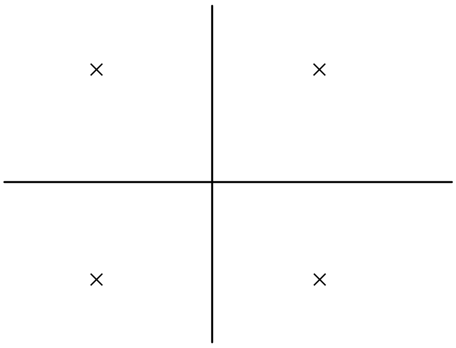
3(a)	$\alpha+\beta+\gamma=2, \beta\gamma+\gamma\alpha+\alpha\beta=2, \alpha\beta\gamma=-1$	**B1**	
	$\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}+\frac{\alpha\beta}{\gamma}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha\beta\gamma}$	**M1**	
	$=\frac{(\beta\gamma+\gamma\alpha+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$	**A1**	
	$=\frac{(2)^2-2\times(-1)\times 2}{-1}=-8$	**A1**	Convincing
(b)	Consider		
	$\frac{\gamma\alpha}{\beta}\times\frac{\alpha\beta}{\gamma}+\frac{\alpha\beta}{\gamma}\times\frac{\beta\gamma}{\alpha}+\frac{\beta\gamma}{\alpha}\times\frac{\gamma\alpha}{\beta}$	**M1**	
	$=\alpha^2+\beta^2+\gamma^2$	**A1**	
	$=(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)$	**A1**	
	$= 4 - 2 \times 2 = 0$	**A1**	
	Consider		
	$\frac{\beta\gamma}{\alpha}\times\frac{\gamma\alpha}{\beta}\times\frac{\alpha\beta}{\gamma}=\alpha\beta\gamma=-1$	**M1A1**	
	The required equation is		
	$x^3+8x^2+1=0$	**B1**	FT their coefficients

4(a)	Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Ref matrix in $y = x$ = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	**M1**	
	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**A1**	
	$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	Fixed points satisfy $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	**M1**	
	$x = x + 1$, $(y = -y + 2)$	**A1**	
	These equations are not consistent so there are no fixed points.	**A1**	Accept equivalent reason
5	Putting $n = 1$, the formula gives 6 which is divisible by 6 so the result is true for $n = 1$	**B1**	
	Assume formula is true for $n = k$, ie $7^k - 1$ is divisible by 6 or $7^k = 6N + 1$	**M1**	
	Consider, for $n = k + 1$, $7^{k+1} - 1 = 7.7^k - 1$	**M1**	
	$= 7(6N + 1) - 1$	**A1**	
	$= 42N + 6$	**A1**	
	This is divisible by 6 therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$, the result is proved by induction.	**A1**	

6(a)(i)	Det(**A**) = $7 - 4\lambda + \lambda(5\lambda - 14) + 3(8 - 5)$ $= 5\lambda^2 - 18\lambda + 16$	**M1** **A1**	
(ii)	Putting $\lambda = 2$, det = 20 – 36 + 16 = 0 So **A** is singular. Putting det(**A**) = 0, product of roots is 16/5 So the other root is 8/5	**B1** **B1**	
(b)(i)	$x + 2y + 3z = 2$ $2x + y + 2z = 1$ $5x + 4y + 7z = 4$		
	Attempting to use row operations, $x + 2y + 3z = 2$ $3y + 4z = 3$ $6y + 8z = 6$ Since the 3rd equation is twice the 2nd equation, it follows that the equations are consistent.	**M1** **A1** **A1** **A1**	Or because the next step gives a row of zeroes
(ii)	Let $z = \alpha$ $y = 1 - \frac{4}{3}\alpha$ $x = -\frac{1}{3}\alpha$ (or equivalent)	**M1** **A1** **A1**	
(c)(i)	**A** = $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ 5 & 4 & 7 \end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix} 3 & -9 & 3 \\ 5 & -8 & 1 \\ -2 & 5 & -1 \end{bmatrix}$ si	**M1A1**	Award M1 if at least 5 correct elements
	Adjugate matrix = $\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}$	**A1**	No FT from incorrect cofactor matrix
(ii)	Determinant = 3	**B1**	
	Inverse matrix = $\frac{1}{3}\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}$	**B1**	FT from incorrect adjugate
(iii)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$	**M1** **A1**	FT from inverse matrix

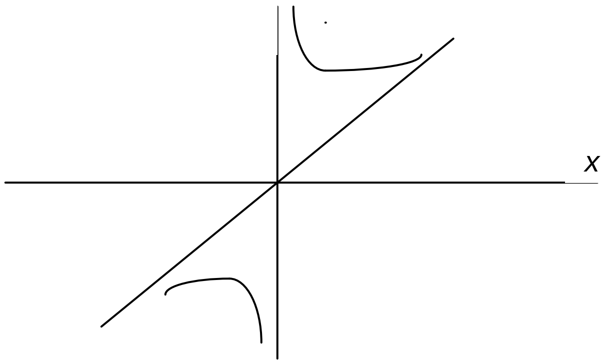
	$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$		
7	Taking logs, $\ln f(x) = \ln\sqrt{1+\sin x} - \ln(1+\tan x)^2$ $= \frac{1}{2}\ln(1+\sin x) - 2\ln(1+\tan x)$ Differentiating, $\frac{f'(x)}{f(x)} = \frac{\cos x}{2(1+\sin x)} - \frac{2\sec^2 x}{(1+\tan x)}$ Putting $x = \pi/4$, $f'(\pi/4) = -0.586$ cao	**M1A1** **A1** **B3** **M1** **A2**	B1 for each correct term
8(a) **(b)**	$u + \mathrm{i}v = (x + \mathrm{i}y)^2$ $= x^2 - y^2 + 2\mathrm{i}xy$ Equating real and imaginary parts, $u = x^2 - y^2$ $v = 2xy$ Substituting for y, $u = x^2 - (2x^2 + 1) = -1 - x^2$ $v^2 = 4x^2(2x^2 + 1)$ Eliminating x, $x^2 = -(u + 1)$ So that $v^2 = 4(u + 1)(2u + 1)$ cao	**M1** **A1** **M1** **A1** **M1** **A1** **A1** **M1** **A1**	 FT their expressions from (a)

Ques	Solution	Mark	Notes
1	$u = x^2 \Rightarrow \mathrm{d}u = 2x\mathrm{d}x$,	**B1**	
	$[1,2] \rightarrow [1, 4]$	**B1**	
	$I = \frac{1}{2}\int_1^4 \frac{\mathrm{d}u}{\sqrt{25-u^2}}$	**M1**	
	$= \frac{1}{2}\left[\sin^{-1}(\frac{u}{5})\right]_1^4$	**A1**	
	$= 0.363$ cao	**A1**	
2(a)	Substituting $t = \tan(\theta/2)$		
	$\frac{2t}{1+t^2} + \frac{3(1-t^2)}{1+t^2} = 2$	**M1A1**	
	$2t + 3 - 3t^2 = 2 + 2t^2$		
	$5t^2 - 2t - 1 = 0$	**A1**	Convincing.
(b)	$t = \frac{2 \pm \sqrt{24}}{10} = 0.68989..,-0.28989..$	**M1A1**	
	t = 0.68989.. giving $\theta/2$ = 0.6039..	**B1**	FT their roots from (a)
	The general solution is θ = 1.21 + 2$n\pi$	**B1**	
	$t = -0.28989..$giving $\theta/2 = -0.2821..$	**B1**	Accept 2.859…
	The general solution is θ = – 0.564 + 2$n\pi$	**B1**	Accept 5.72 + 2$n\pi$

3(a)	$-1 = \cos\pi + i\sin\pi$	**B1**	
	$\sqrt[4]{-1} = \cos\pi/4 + i\sin\pi/4 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	**M1A1**	
	$\text{Root2} = \cos 3\pi/4 + i\sin 3\pi/4 = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	**A1**	
	$\text{Root3} = \cos 5\pi/4 + i\sin 5\pi/4 = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	**A1**	Special case : Award 2/6 if they misread –1 as 1.
	$\text{Root4} = \cos 7\pi/4 + i\sin 7\pi/4 = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	**A1**	
(b)(i)		**B1**	FT their roots if possible
(ii)	Length of side = $\frac{2}{\sqrt{2}}$	**B1**	
	Area of square = 2	**B1**	

4(a)	$f'(x)=\dfrac{2(x-1)-(2x+3)}{(x-1)^2}$ $=-\dfrac{5}{(x-1)^2}$ This is negative for all $x > 1$ therefore f is strictly decreasing.	**M1** **A1** **A1**	
(b)(i)	$f(4) = 11/3$, $f(5) = 13/4$ $f(S) = [13/4,11/3]$	**M1** **A1**	A0 if wrong way around but penalise only once.
(ii)	EITHER $y=\dfrac{2x+3}{x-1} \Rightarrow x=\dfrac{y+3}{y-2}$ $f^{-1}(4)=7/2,\ f^{-1}(5)=8/3$ $f^{-1}(S)=[8/3,7/2]$ OR $\dfrac{2x+3}{x-1}=4 \rightarrow x=\dfrac{7}{2}$ $\dfrac{2x+3}{x-1}=5 \rightarrow x=\dfrac{8}{3}$ $f^{-1}(S)=[8/3,7/2]$	**M1A1** **A1** **A1** **M1A1** **A1** **A1**	 A0 if wrong way around. M1A1 for the first and then A1 for the second. A0 if wrong way around.
5(a)(i)	Completing the square, $(x-2)^2+2(y+1)^2=4$ The centre is therefore (2, –1)	**M1A1** **A1**	
(ii)	In standard form, the equation is $\dfrac{(x-2)^2}{4}+\dfrac{(y+1)^2}{2}=1$ so $a = 2$, $b = \sqrt{2}$ si $e=\sqrt{\dfrac{4-2}{4}}=\dfrac{1}{\sqrt{2}}$	**B1** **M1A1**	FT their equation in (ii), (iii) and (iv)
(iii)	The foci are $\left(2+\sqrt{2},-1\right)$ and $\left(2-\sqrt{2},-1\right)$	**B1B1**	
(iv)	The equations of the directrices are $x=2\pm2\sqrt{2}$	**B1**	
(b)(i)	EITHER Putting $x = 0$, $(y+1)^2=0$ This has a repeated root, hence $x = 0$ is a tangent OR Semi-major axis = 2 = x-coordinate of centre This equality shows that $x = 0$ is a tangent	 **M1** **A1** **M1** **A1**	
(ii)	Substituting $y = mx$, $x^2(1+2m^2)-x(4-4m)+2=0$ Use of the condition for tangency, ie '$b^2=4ac$' $16(1-m)^2=8(1+2m^2)$ $2-4m+2m^2=1+2m^2 \Rightarrow m=\dfrac{1}{4}$	**M1** **A1** **M1** **A1** **A1**	

6(a)	Let $\dfrac{4x^2-2x+9}{x(x^2+3)} \equiv \dfrac{A}{x}+\dfrac{Bx+C}{x^2+3}$		
	$= \dfrac{A(x^2+3)+x(Bx+C)}{x(x^2+3)}$ (oe)	**M1**	
	$x = 0$ gives $A = 3$	**A1**	
	Coeff of x^2 gives $A + B = 4$, $B = 1$	**A1**	
	Coeff of x gives $C = -2$	**A1**	
(b)	$\displaystyle\int_1^3 \frac{4x^2-2x+9}{x(x^2+3)}\,\mathrm{d}x = \int_1^3\left(\frac{3}{x}+\frac{x}{x^2+3}-\frac{2}{x^2+3}\right)\mathrm{d}x$	**M1**	
	$= \left[3\ln x+\dfrac{1}{2}\ln(x^2+3)-\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)\right]_1^3$	**B3**	B1 each term
	$= 3\ln 3+\dfrac{1}{2}\ln 12-\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{3}{\sqrt{3}}\right)$ $-3\ln 1-\dfrac{1}{2}\ln 4+\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$	**A1**	
	= 3.24 cao	**A1**	

Question	Solution	Mark	Notes
7(a)	Consider $f(-x) = \frac{(2(-x)^2+1)^2}{(-x)^3} = -f(x)$	**M1A1**	
	Therefore f is odd	**A1**	
(b)	EITHER Differentiating, $f'(x) = \frac{2(2x^2+1).4x.x^3 - 3x^2(2x^2+1)^2}{x^6}$	**M1A1**	
	At a stationary point, putting $f'(x)=0$, $8x^2 = 3(2x^2+1)$	**m1**	Condone the cancellation of $x^2(2x^2+1)$
	$x = \pm\sqrt{\frac{3}{2}}$	**A1**	
	OR Consider $f(x) = 4x + \frac{4}{x} + \frac{1}{x^3}$	**M1**	
	$f'(x) = 4 - \frac{4}{x^2} - \frac{3}{x^4}$	**A1**	
	At a stationary point, putting $f'(x)=0$, $4x^4 - 4x^2 - 3 = 0$	**m1**	
	$x = \pm\sqrt{\frac{3}{2}}$	**A1**	
(c)	The asymptotes are $x = 0$ $y = 4x$	**B1** **B1**	
(d)		**G1** **G1**	

8	EITHER Consider $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$ Expanding and taking real parts, $\cos 5\theta = \cos^5\theta + 10\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+ 5\cos\theta(\mathrm{i}\sin\theta)^4$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta$ $-10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**M1** **m1A1** **A1** **A1** **A1**	
	OR Let $z = \cos\theta + \mathrm{i}\sin\theta$ So that $z + \frac{1}{z} = 2\cos\theta$ and $z^n + \frac{1}{z^n} = 2\cos n\theta$ Consider $\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$ $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$ $= 16\cos^5\theta - 5(4\cos^3\theta - 3\cos\theta) - 10\cos\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**M1** **A1** **A1** **A1** **A1** **A1**	

Ques	Solution	Mark	Notes
1	Using $\cosh 2x = 2\cosh^2 x - 1$, the eqn becomes $2\cosh^2 x - 7\cosh x + 6 = 0$ Solving the quadratic equation, $\cosh x$ = 2, 1.5 The positive roots are therefore $x = \cosh^{-1} 2 = 1.32$ and $x = \cosh^{-1}(1.5) = 0.96$	M1 A1 M1 A1 A1 A1	 FT their roots
2(a)(i)	The Newton-Raphson iteration is $x_{n+1} = x_n - \dfrac{(x_n{}^3 - a)}{3x_n^2}$ $= \dfrac{2x_n^3 + a}{3x_n^2}$	 M1 A1	 Convincing
(ii)	$x_0 = 2$ $x_1 = 2.166666667$ $x_2 = 2.154503616$ $x_3 = 2.154434692$ $x_4 = 2.15443469$ $\sqrt[3]{10} = 2.1544$ correct to 4 decimal places.	 M1A1 A1	
(b)	Consider $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{a}{x^2}\right) = -\dfrac{2a}{x^3}$ $= -2$ when $x = \sqrt[3]{a}$ The sequence diverges because this exceeds 1 in modulus.	 M1A1 A1 A1	 M0 if a =10
3(a)	$f'(x) = \dfrac{2\mathrm{e}^x}{2\mathrm{e}^x - 1}$ $f''(x) = \dfrac{2\mathrm{e}^x(2\mathrm{e}^x - 1) - 2\mathrm{e}^x.2\mathrm{e}^x}{(2\mathrm{e}^x - 1)^2}$ $= \dfrac{-2\mathrm{e}^x}{(2\mathrm{e}^x - 1)^2}$	B1 M1 A1	 convincing
(b)	$f'''(x) = \dfrac{-2\mathrm{e}^x(2\mathrm{e}^x - 1)^2 + 2\mathrm{e}^x.2\mathrm{e}^x 2(2\mathrm{e}^x - 1)}{(2\mathrm{e}^x - 1)^4}$ $f(0) = 0, f'(0) = 2, f''(0) = -2, f'''(0) = 6$ The Maclaurin series is $2x - x^2 + x^3 + \ldots$	M1A1 B2 M1A1	 Award B1 for 2 correct values FT on their values of $f^{(n)}(0)$

4	Completing the square, $3+2x-x^2=4-(x-1)^2$ so $I=\int_1^2\sqrt{4-(x-1)^2}\,\mathrm{d}x$ Put $x-1=2\sin\theta$ $\mathrm{d}x = 2\cos\theta\mathrm{d}\theta$, $[1,2] \to [0,\pi/6]$ $I=\int_0^{\pi/6}\sqrt{4-4\sin^2\theta}\,.2\cos\theta\mathrm{d}\theta$ $= 4\int_0^{\pi/6}\cos^2\theta\,\mathrm{d}\theta$ $= 2\int_0^{\pi/6}(1+\cos 2\theta)\,\mathrm{d}\theta$ $= 2\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/6}$ $= 1.91$	M1A1 M1 A1A1 m1 A1 A1 A1 A1	Allow $x-1=2\cos\theta$
5(a) **(b)**	$I_n=\left[x^n\cosh x\right]_0^1-n\int_0^1 x^{n-1}\cosh x\mathrm{d}x$ $= \cosh 1-n\int_0^1 x^{n-1}\cosh x\mathrm{d}x$ $=\cosh 1-\left[nx^{n-1}\sinh x\right]_0^1+n(n-1)I_{n-2}$ $= \cosh 1-n\sinh 1+n(n-1)I_{n-2}$ $I_0=\int_0^1\sinh x\mathrm{d}x=\left[\cosh x\right]_0^1=\cosh 1-1$ $I_4=\cosh 1-4\sinh 1+12I_2$ $= \cosh 1-4\sinh 1+12(\cosh 1-2\sinh 1+2I_0)$ $= 13\cosh 1 - 28\sinh 1 + 24(\cosh 1-1)$ $= 37\cosh 1 - 28\sinh 1 - 24$ cao	M1A1 A1 M1A1 M1A1 M1 A1 A1	 M1A1 for evaluating I_0 at any stage FT their I_0 if substituted here

6(a)	Consider $x = r\cos\theta$ $= \sin^2\theta\cos\theta$ $\frac{dx}{d\theta} = 2\sin\theta\cos^2\theta - \sin^3\theta$ The tangent is perpendicular to the initial line where $\frac{dx}{d\theta} = 2\sin\theta\cos^2\theta - \sin^3\theta = 0$ $\tan^2\theta = 2$ $\theta = \tan^{-1}\sqrt{2} = 0.955$ $r = 0.667$	 M1 A1 M1A1 M1 A1 A1 A1	 Do not penalise the removal of the factor sinθ
(b)	Area = $\frac{1}{2}\int r^2 d\theta$ $= \frac{1}{2}\int_0^{\pi/2}(1-\sin\theta)^2 d\theta$ $= \frac{1}{2}\int_0^{\pi/2}(1-2\sin\theta+\sin^2\theta)d\theta$ $= \frac{1}{4}\int_0^{\pi/2}(3-4\sin\theta-\cos 2\theta)d\theta$ $= \frac{1}{4}\left[3\theta+4\cos\theta-\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$ $= \frac{3\pi-8}{8}$ (0.178) cao	M1 A1 A1 A1 A1 A1	

7(a)(i)	$D(\text{cosech}x) = D\left(\frac{1}{\sinh x}\right)$	M1	
	$= \frac{-1}{\sinh^2 x} \times \cosh x$	A1	
	$= -\text{cosech}x\coth x$		
	$D(\coth x) = D\left(\frac{\cosh x}{\sinh x}\right)$	M1	
	$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$	A1	
	$= -\text{cosech}^2 x$		
(ii)	$D\ln(\text{cosech}x + \coth x)$		
	$= \frac{-(\text{cosech}x\coth x + \text{cosech}^2 x)}{(\text{cosech}x + \coth x)}$	M1	
	$= -\text{cosech}x$	A1	convincing
(b)(i)	$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	M1	
	$= \int_1^e \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx$	A1	
	$= \int_1^e \frac{\sqrt{1+x^2}}{x}\, dx$		
(ii)	Putting $x = \sinh u$,		
	$dx = \cosh u\,du$, $[1,e] \to [\sinh^{-1}1, \sinh^{-1}e]$ $([\alpha,\beta])$	B1B1	
	Arc length $= \int_\alpha^\beta \frac{\sqrt{1+\sinh^2 u}}{\sinh u}.\cosh u\,du$	M1	
	$= \int_\alpha^\beta \frac{\cosh^2 u}{\sinh u} du$	A1	
	$= \int_\alpha^\beta \frac{1+\sinh^2 u}{\sinh u} du$	A1	
	$= \int_\alpha^\beta (\text{cosech}u + \sinh u) du$		
(iii)		M1A1	
	$= \left[-\ln(\text{cosech}u + \coth u) + \cosh u\right]_\alpha^\beta$	A2	
	$= 2.00$		

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