



# **GCE MARKING SCHEME**

**MATHEMATICS  
AS/Advanced**

**JANUARY 2014**

## **INTRODUCTION**

The marking schemes which follow were those used by WJEC for the January 2014 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

<b>Unit</b>	<b>Page</b>
C1	1
C2	6
C3	11
M1	17
S1	26
FP1	29

# Mathematics C1 January 2014

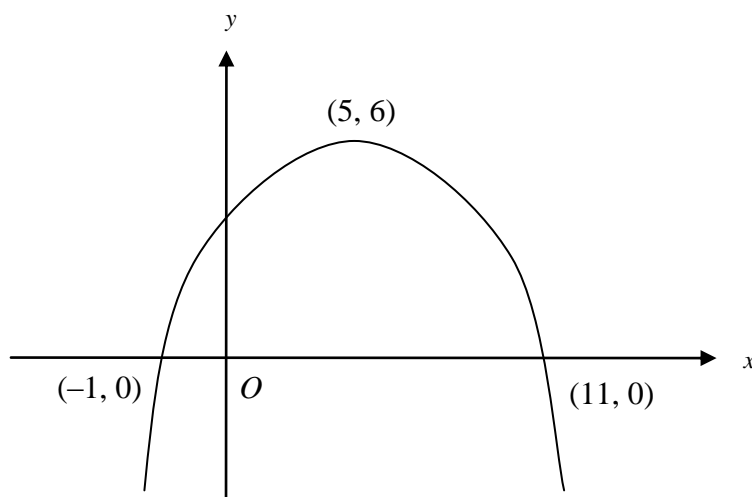
## Solutions and Mark Scheme

### Final Version

1. (a) (i) Gradient of  $AB = \frac{\text{increase in } y}{\text{increase in } x}$  M1  
 Gradient of  $AB = -\frac{3}{2}$  (or equivalent) A1
- (ii) Use of gradient  $L_1 \times \text{gradient } AB = -1$  M1  
 A correct method for finding the equation of  $L_1$  using candidate's gradient for  $L_1$  M1  
 Equation of  $L_1: y - 1 = \frac{2}{3}(x - 4)$  (or equivalent) A1  
 (f.t. candidate's gradient for  $AB$ )
- (b) (i) An attempt to solve equations of  $L_1$  and  $L_2$  simultaneously M1  
 $x = -2, y = -3$  (convincing) A1
- (ii) A correct method for finding the coordinates of the mid-point of  $AC$  M1  
 Mid-point of  $AC$  has coordinates  $(2, -2.5)$  (c.a.o.) A1
- (iii) A correct method for finding the length of  $AB(BC)$  M1  
 $AB = \sqrt{13}$  A1  
 $BC = \sqrt{52}$  (or equivalent) A1  
 A correct method for finding the area of triangle  $ABC$  m1  
 Area of triangle  $ABC = 13$  (c.a.o.) A1
2.  $\frac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = \frac{(3\sqrt{3} - 2\sqrt{5})(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})}$  M1  
 Numerator:  $6 \times 3 - 3 \times \sqrt{3} \times \sqrt{5} - 4 \times \sqrt{5} \times \sqrt{3} + 10$  A1  
 Denominator:  $12 - 5$  A1  
 $\frac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = 4 - \sqrt{15}$  (c.a.o.) A1
- Special case**  
 If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by  $2\sqrt{3} + \sqrt{5}$

3. An attempt to differentiate, at least one non-zero term correct M1  
 $\frac{dy}{dx} = 20 \times -1 \times x^{-2} + 4x$  A1  
 An attempt to substitute  $x = 2$  in candidate's derived expression for  $\frac{dy}{dx}$  m1  
 Value of  $\frac{dy}{dx}$  at  $P = 3$  (c.a.o.) A1  
 Gradient of normal =  $\frac{-1}{\text{candidate's derived value for } \frac{dy}{dx}}$  m1  
 Equation of normal to  $C$  at  $P$ :  $y - 7 = -\frac{1}{3}(x - 2)$  (or equivalent)  
 (f.t. candidate's value for  $\frac{dy}{dx}$  provided all three method marks are awarded) A1
4. Either  $p = 0.8$  or a sight of  $(x + 0.8)^2$  B1  
 A convincing argument to show that the value 25 is correct B1  
 $x^2 + 1.6x - 24.36 = 0 \Rightarrow (x + 0.8)^2 = 25$  (f.t. candidate's value for  $p$ ) M1  
 $x = 4.2$  (f.t. candidate's value for  $p$ ) A1  
 $x = -5.8$  (f.t. candidate's value for  $p$ ) A1
5. (a)  $(1 + \sqrt{6})^5 = (1)^5 + 5(1)^4(\sqrt{6}) + 10(1)^3(\sqrt{6})^2 + 10(1)^2(\sqrt{6})^3 + 5(1)(\sqrt{6})^4 + (\sqrt{6})^5$  (five or six terms correct) B2  
 (If B2 not awarded, award B1 for four correct terms)  
 $(1 + \sqrt{6})^5 = 1 + 5\sqrt{6} + 60 + 60\sqrt{6} + 180 + 36\sqrt{6}$  (six terms correct) B2  
 (If B2 not awarded, award B1 for four or five correct terms)  
 $(1 + \sqrt{6})^5 = 241 + 101\sqrt{6}$  (f.t. one error) B1
- (b)  ${}^nC_2 \times 3^k = 495$  ( $k = 1, 2$ ) M1  
 Either  $9n^2 - 9n - 990 = 0$  or  $n^2 - n - 110 = 0$  or  $n(n - 1) = 110$  A1  
 $n = 11$  (c.a.o.) A1
6. An expression for  $b^2 - 4ac$ , with at least two of  $a, b, c$  correct M1  
 $b^2 - 4ac = 8^2 - 4 \times (2k - 3) \times (2k + 3)$  A1  
 Putting  $b^2 - 4ac < (\leq) 0$  m1  
 $100 - 16k^2 < 0$  (o.e.) (c.a.o.) A1  
 Finding critical values  $k = -5/2, k = 5/2$   
 (o.e.) (f.t. candidate's values for  $m, n$ ) B1  
 $k < -5/2$  or  $5/2 < k$  (o.e.) (f.t. only critical values of  $-a$  and  $a$ ) B1  
 Each of the following errors earns a final B0  
 the use of non-strict inequalities  
 the use of the word 'and' instead of the word 'or'

7. (a)



Concave down curve and y-coordinate of maximum = 6      B1  
 x-coordinate of maximum = 5      B1  
 Both points of intersection with x-axis      B1

(b)  $y = f(-2x)$       B2  
 (If B2 not awarded, award B1 for either  $y = f(-\frac{1}{2}x)$  or  $y = f(2x)$ )

8. (a)  $y + \delta y = 7(x + \delta x)^2 - 6(x + \delta x) - 3$       B1  
 Subtracting  $y$  from above to find  $\delta y$       M1  
 $\delta y = 14x\delta x + 7(\delta x)^2 - 6\delta x$       A1  
 Dividing by  $\delta x$  and letting  $\delta x \rightarrow 0$       M1  
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 14x - 6$       (c.a.o.) A1

(b)  $\frac{dy}{dx} = a \times \frac{4}{3} \times x^{1/3} + 24 \times \frac{1}{2} \times x^{-1/2}$       B1, B1  
 Attempting to substitute  $x = 64$  in candidate's expression for  $\frac{dy}{dx}$   
 putting expression equal to  $\frac{11}{2}$       M1  
 (The M1 is only awarded if at least one B1 has been awarded)  
 $a = \frac{3}{4}$       (c.a.o.) A1

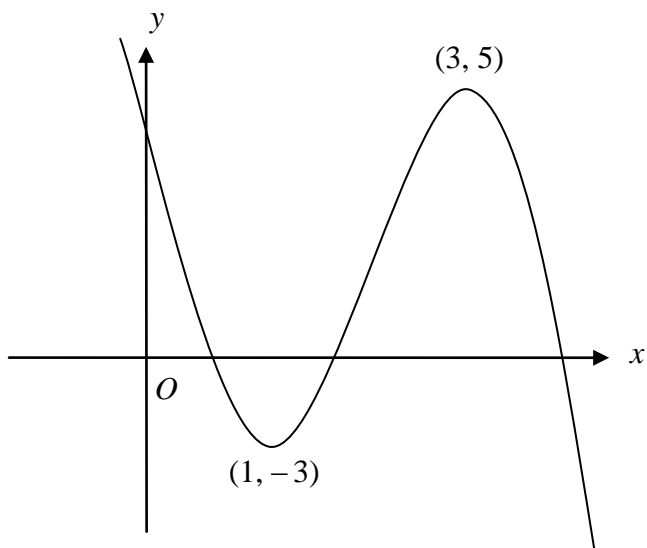
9. (a) Use of  $f(-3) = -39$  M1  
 $-27a + 117 + 30 - 24 = -39 \Rightarrow a = 6$  (convincing) A1
- (b) Attempting to find  $f(r) = 0$  for some value of  $r$  M1  
 $f(-2) = 0 \Rightarrow x + 2$  is a factor A1  
 $f(x) = (x + 2)(6x^2 + ax + b)$  with one of  $a, b$  correct M1  
 $f(x) = (x + 2)(6x^2 + x - 12)$  A1  
 $f(x) = (x + 2)(2x + 3)(3x - 4)$  (f.t. only  $6x^2 - x - 12$  in above line) A1  
 $x = -2, -\frac{3}{2}, \frac{4}{3}$  (f.t. for factors  $2x \pm 3, 3x \pm 4$ ) A1

**Special case**

Candidates who, after having found  $x + 2$  as one factor, then find just one of the remaining factors by using e.g. the factor theorem, are awarded B1 for the final 4 marks

10. (a)  $\frac{dy}{dx} = -6x^2 + 24x - 18$  B1  
 Putting derived  $\frac{dy}{dx} = 0$  M1  
 $x = 1, 3$  (both correct) (f.t. candidate's  $\frac{dy}{dx}$ ) A1  
 Stationary points are  $(1, -3)$  and  $(3, 5)$  (both correct) (c.a.o) A1  
 A correct method for finding nature of stationary points yielding  
**either**  $(1, -3)$  is a minimum point  
**or**  $(3, 5)$  is a maximum point (f.t. candidate's derived values) M1  
 Correct conclusion for other point (f.t. candidate's derived values) A1

(b)



- Graph in shape of a negative cubic with two turning points M1  
 Correct marking of both stationary points  
 (f.t. candidate's derived maximum and minimum points) A1
- (c) Use of both  $k = -3, k = 5$  to find the range of values for  $k$   
 (f.t. candidate's y-values at stationary points) M1  
 $-3 < k < 5$  (f.t. candidate's y-values at stationary points) A1

# Mathematics C2 January 2014

## Solutions and Mark Scheme

### Final Version

1.	2	2		
	2.5	1.843908891		
	3	1.732050808		
	3.5	1.647508942		
	4	1.58113883	(5 values correct)	B2
	<b>(If B2 not awarded, award B1 for either 3 or 4 values correct)</b>			

Correct formula with  $h = 0.5$  M1

$$I \approx \frac{0.5}{2} \times \{2 + 1.58113883 + 2(1.843908891 + 1.732050808 + 1.647508942)\}$$

$$I \approx 14.02807611 \times 0.5 \div 2$$

$$I \approx 3.507019028$$

$$I \approx 3.507 \quad (\text{f.t. one slip}) \quad \text{A1}$$

**Special case** for candidates who put  $h = 0.4$

2	2		
2.4	1.870828693		
2.8	1.772810521		
3.2	1.695582496		
3.6	1.632993162		
4	1.58113883	(all values correct)	B1

Correct formula with  $h = 0.4$  M1

$$I \approx \frac{0.4}{2} \times \{2 + 1.58113883 + 2(1.870828693 + 1.772810521 + 1.695582496 + 1.632993162)\}$$

$$I \approx 17.52556857 \times 0.4 \div 2$$

$$I \approx 3.505113715$$

$$I \approx 3.505 \quad (\text{f.t. one slip}) \quad \text{A1}$$

**Note: Answer only with no working earns 0 marks**



2. (a)  $8 \cos^2 \theta - 7(1 - \cos^2 \theta) = 4 \cos \theta - 3$  (correct use of  $\sin^2 \theta = 1 - \cos^2 \theta$ ) M1  
 An attempt to collect terms, form and solve quadratic equation in  $\cos \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cos \theta + b)(c \cos \theta + d)$ , with  $a \times c =$  candidate's coefficient of  $\cos^2 \theta$  and  $b \times d =$  candidate's constant m1  
 $15 \cos^2 \theta - 4 \cos \theta - 4 = 0 \Rightarrow (5 \cos \theta + 2)(3 \cos \theta - 2) = 0$   
 $\Rightarrow \cos \theta = \frac{2}{3}, \cos \theta = -\frac{2}{5}$  (c.a.o.) A1  
 $\theta = 48.19^\circ, 311.81^\circ$  B1  
 $\theta = 113.58^\circ, 246.42^\circ$  B1 B1  
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.  
 $\cos \theta = +, -, \text{ f.t. for 3 marks, } \cos \theta = -, -, \text{ f.t. for 2 marks}$   
 $\cos \theta = +, +, \text{ f.t. for 1 mark}$
- (b)  $X = 114^\circ$  B1  
 $Y - Z = 20^\circ$  B1  
 $114^\circ + Y + Z = 180^\circ$  (f.t. only for an obtuse value for X) M1  
 $Y = 43^\circ, Z = 23^\circ$  (f.t. one error) A1
3. (a)  $a + 2d + a + 7d = 0$  B1  
 $a + 4d + a + 6d + a + 9d = 22$  B1  
 An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1  
 $a = -18, d = 4$  (both values) (c.a.o.) A1
- (b)  $S_n = \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$  B1  
 $S_{2n} = \frac{2n}{2}[2 \times 9 + (2n - 1) \times 2]$  B1  
 $\frac{2n}{2}[2 \times 9 + (2n - 1) \times 2] = k \times \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$  ( $k = 3, \frac{1}{3}$ )  
 (f.t. candidate's quadratic expressions for  $S_{2n}, S_n$  provided at least one of the first two B marks is awarded) M1  
 An attempt to solve this equation including dividing both sides by  $n$  to reach a linear equation in  $n$ . m1  
 $n = 8$  (c.a.o.) A1

4. (a)  $S_n = a + ar + \dots + ar^{n-1}$  (at least 3 terms, one at each end) B1  
 $rS_n = ar + \dots + ar^{n-1} + ar^n$   
 $S_n - rS_n = a - ar^n$  (multiply first line by  $r$  and subtract) M1  
 $(1-r)S_n = a(1-r^n)$   
 $S_n = \frac{a(1-r^n)}{1-r}$  (convincing) A1
- (b) (i)  $ar^3 = -108$  and  $ar^6 = 4$  B1  
 $r^3 = \frac{4}{-108}$  (o.e.) M1  
 $r = -\frac{1}{3}$  (c.a.o.) A1
- (ii)  $a \times (-\frac{1}{3})^3 = -108 \Rightarrow a = 2916$  (f.t. candidate's derived value for  $r$ ) B1  
 $S_\infty = \frac{2916}{1 - (-\frac{1}{3})}$  (use of formula for sum to infinity)  
(f.t. candidate's derived values for  $r$  and  $a$ ) M1  
 $S_\infty = 2187$  (c.a.o.) A1
5. (a) (i) **Either:**  $5^2 = 3^2 + x^2 - 2 \times 3 \times x \times \cos ADB$  (o.e.)  
**Or:**  $6^2 = 1^2 + x^2 - 2 \times 1 \times x \times \cos ADC$  (o.e.)  
(at least one correct use of cos rule) M1  
 $\cos ADB = \frac{x^2 - 16}{6x}$  (convincing) A1  
 $\cos ADC = \frac{x^2 - 35}{2x}$  A1
- (ii)  $\frac{x^2 - 16}{6x} + \frac{x^2 - 35}{2x} = 0$  (o.e.)  
(f.t. candidate's derived expression for  $\cos ADC$ ) M1  
 $4x^2 = 121$  (f.t. candidate's derived expression for  $\cos ADC$  providing it is of similar form) A1  
 $x = 5.5$  (convincing) (c.a.o.) A1
- (b)  $ADB = 64.42^\circ$  B1  
Area of triangle  $ADB = \frac{5.5 \times 3 \times \sin 64.42^\circ}{2}$   
(f.t. candidate's derived value for angle  $ADB$ ) M1  
Area of triangle  $ADB = 7.44 \text{ cm}^2$  (c.a.o.) A1

6. (a)  $5 \times \frac{x^{-2}}{-2} - 2 \times \frac{x^{4/3}}{4/3} - 4x + c$  B1, B1, B1  
 (–1 if no constant term present)

(b) Area =  $\int_2^6 \left( 3x^2 - \frac{1}{4}x^3 \right) dx$  (use of integration) M1

$\frac{3x^3}{3} - \frac{1}{4 \times 4} x^4$  (correct integration) B1

Area =  $(216 - 81) - (8 - 1)$  (correct method for substituting limits) m1

Area = 128 (c.a.o.) A1

7. (a) Let  $p = \log_a x$   
 Then  $x = a^p$  (relationship between log and power) B1  
 $x^n = a^{pn}$  (the laws of indices) B1  
 $\therefore \log_a x^n = pn$  (relationship between log and power)  
 $\therefore \log_a x^n = pn = n \log_a x$  (convincing) B1

(b) **Either:**  
 $(5 - 4x) \log_{10} 7 = \log_{10} 11$   
 (taking logs on both sides and using the power law) M1

$x = \frac{5 \log_{10} 7 - \log_{10} 11}{4 \log_{10} 7}$  A1

$x = 0.942$  (f.t. one slip, see below) A1

**Or:**  
 $5 - 4x = \log_7 11$  (rewriting as a log equation) M1

$x = \frac{5 - \log_7 11}{4}$  A1

$x = 0.942$  (f.t. one slip, see below) A1

Note: an answer of  $x = -0.942$  from  $x = \frac{\log_{10} 11 - 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

an answer of  $x = 1.558$  from  $x = \frac{\log_{10} 11 + 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

**Note: Answer only with no working shown earns 0 marks**

(c)  $\log_8 x = -\frac{1}{3} \Rightarrow x = 8^{-1/3}$  (rewriting log equation as power equation) M1

$x = 8^{-1/3} \Rightarrow x = \frac{1}{2}$  A1

8. (a) (i)  $A(2, -4)$  B1  
(ii) Gradient  $AP = \frac{\text{inc in } y}{\text{inc in } x}$  M1  
Gradient  $AP = \frac{(-7) - (-4)}{6 - 2} = -\frac{3}{4}$   
(f.t. candidate's coordinates for A) A1  
Use of  $m_{\text{tan}} \times m_{\text{rad}} = -1$  M1  
Equation of tangent is:  
 $y - (-7) = \frac{4}{3}(x - 6)$  (f.t. candidate's gradient for AP) A1
- (b) An attempt to substitute  $(x + 3)$  for  $y$  in the equation of the circle and form quadratic in  $x$  M1  
 $x^2 + (x + 3)^2 - 4x + 8(x + 3) - 5 = 0 \Rightarrow 2x^2 + 10x + 28 = 0$  A1  
An attempt to calculate value of discriminant m1  
Discriminant =  $100 - 224 < 0 \Rightarrow$  no points of intersection  
(f.t. one slip) A1
9. Denoting  $\widehat{AOB}$  by  $\theta$ ,  
Area of sector  $AOB = \frac{1}{2} \times 7^2 \times \theta$   
Area of sector  $COD = \frac{1}{2} \times 4^2 \times \theta$  (at least one correct) M1  
 $\frac{1}{2} \times 7^2 \times \theta - \frac{1}{2} \times 4^2 \times \theta = 23 \cdot 1$   
 $\theta = 1.4$  (f.t candidate's expressions for the areas of the sectors) m1  
(c.a.o.)  
A1  
 $CD = 5.6 \text{ cm}, AB = 9.8 \text{ cm}$  (both values, f.t candidate's value for  $\theta$ ) B1  
Use of perimeter of  $ACDB = AC + CD + DB + BA$  M1  
Perimeter of  $ACDB = 21.4 \text{ cm}$  (c.a.o.) A1
10. (a)  $t_2 = \frac{3}{4}$  B1  
 $t_3 = -\frac{1}{3}, t_4 = 4$  B1
- (b) The sequence repeats itself every third term B1  
 $t_{50} = \frac{3}{4}$  B1

# Mathematics C3 January 2014

## Solutions and Mark Scheme

### Final Version

1. (a) 0 0  
 $\pi/12$  0.071796769  
 $\pi/6$  0.333333333  
 $\pi/4$  1  
 $\pi/3$  3 (5 values correct) B2  
(If B2 not awarded, award B1 for either 3 or 4 values correct)  
Correct formula with  $h = \pi/12$  M1  
 $I \approx \frac{\pi/12}{3} \times \{0 + 3 + 4(0.071796769 + 1) + 2(0.333333333)\}$   
 $I \approx 7.953853742 \times (\pi/12) \div 3$   
 $I \approx 0.69410468$   
 $I \approx 0.6941$  (f.t. one slip) A1

**Note: Answer only with no working shown earns 0 marks**

- (b)  $\int_0^{\pi/3} \sec^2 x \, dx = \int_0^{\pi/3} 1 \, dx + \int_0^{\pi/3} \tan^2 x \, dx$  M1  
 $\int_0^{\pi/3} \sec^2 x \, dx = 1.7413$  (f.t. candidate's answer to (a)) A1

**Note: Answer only with no working shown earns 0 marks**

2. (a) Choice of  $x$  satisfying  $75^\circ \leq x < 90^\circ$  and one correct evaluation B1  
Both evaluations correct B1
- (b)  $15(1 + \cot^2 \theta) + 2 \cot \theta = 23$   
(correct use of  $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ ) M1  
An attempt to collect terms, form and solve quadratic equation in  $\cot \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cot \theta + b)(c \cot \theta + d)$ , with  $a \times c =$  candidate's coefficient of  $\cot^2 \theta$  and  $b \times d =$  candidate's constant m1  
 $15 \cot^2 \theta + 2 \cot \theta - 8 = 0 \Rightarrow (5 \cot \theta + 4)(3 \cot \theta - 2) = 0$   
 $\Rightarrow \cot \theta = \frac{2}{3}, \cot \theta = -\frac{4}{5}$   
 $\Rightarrow \tan \theta = \frac{3}{2}, \tan \theta = -\frac{5}{4}$  (c.a.o.) A1  
 $\theta = 56.31^\circ, 236.31^\circ$  B1  
 $\theta = 128.66^\circ, 308.66^\circ$  B1 B1  
Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.  
 $\tan \theta = +, -, \text{ f.t. for 3 marks, } \tan \theta = -, -, \text{ f.t. for 2 marks}$   
 $\tan \theta = +, +, \text{ f.t. for 1 mark}$

3.  $\frac{d(x^3)}{dx} = 3x^2$   $\frac{d(3)}{dx} = 0$  B1  
 $\frac{d(-2x^2y)}{dx} = -2x^2 \frac{dy}{dx} - 4xy$  B1  
 $\frac{d(3y^2)}{dx} = 6y \frac{dy}{dx}$  B1  
 $\frac{dy}{dx} = \frac{-4}{-14} = \frac{2}{7}$  (c.a.o.) B1

4. (a)  $\frac{dx}{dt} = 6t^2$  B1
- (b)  $\frac{d}{dt} \left[ \frac{dy}{dx} \right] = 2 + 12t^2$  B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left[ \frac{dy}{dx} \right] \div \frac{dx}{dt}$  M1  
 $\frac{d^2y}{dx^2} = \frac{2 + 12t^2}{6t^2}$  (c.a.o.) A1  
 $\frac{d^2y}{dx^2} = 2 \Rightarrow 2 + 12t^2 = 12t^2 (\Rightarrow 2 = 0) \Rightarrow$  no such  $t$  exists E1
- (c) Use of  $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$  M1  
 $\frac{dy}{dt} = 12t^3 + 24t^5$  (f.t. candidate's expression for  $\frac{dx}{dt}$ ) A1  
 Use of a valid method of integration to find  $y$  m1  
 $y = 3t^4 + 4t^6 (+ c)$  (f.t. one error in candidate's  $\frac{dy}{dt}$ ) A1  
 $y = 3t^4 + 4t^6 + 3$  (c.a.o.) A1
5.  $x_0 = 1$   
 $x_1 = 0.612372435$  ( $x_1$  correct, at least 5 places after the point) B1  
 $x_2 = 0.62777008$   
 $x_3 = 0.627136142$   
 $x_4 = 0.627162204 = 0.62716$  ( $x_4$  correct to 5 decimal places) B1  
 Let  $h(x) = x^3 + 7x^2 - 3$   
 An attempt to check values or signs of  $h(x)$  at  $x = 0.627155$ ,  
 $x = 0.627165$  M1  
 $h(0.627155) = -6.15 \times 10^{-5} < 0$ ,  $h(0.627165) = 3.81 \times 10^{-5} > 0$  A1  
 Change of sign  $\Rightarrow \alpha = 0.62716$  correct to five decimal places A1

6. (a)  $\frac{dy}{dx} = 10 \times (5x^3 - x)^9 \times f(x)$   $(f(x) \neq 1)$  M1  
 $\frac{dy}{dx} = 10(5x^3 - x)^9(15x^2 - 1)$  A1
- (b) **Either**  $\frac{dy}{dx} = \frac{f(x)}{\sqrt{1 - (x^3)^2}}$  (including  $f(x) = 1$ ) **or**  $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1 - x^5}}$  M1  
 $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1 - x^6}}$  A1
- (c)  $\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$  M1  
 $\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$  (either  $f(x) = 2 \times \frac{1}{2x}$  or  $g(x) = 4x^3$ ) A1  
 $\frac{dy}{dx} = x^3 + 4x^3 \ln(2x)$  (all correct) A1
- (d)  $\frac{dy}{dx} = \frac{(2x + 3)^6 \times k \times e^{4x} - e^{4x} \times 6 \times (2x + 3)^5 \times m}{[(2x + 3)^6]^2}$   
with either  $k = 4, m = 2$  or  $k = 4, m = 1$  or  $k = 1, m = 2$  M1  
 $\frac{dy}{dx} = \frac{(2x + 3)^6 \times 4 \times e^{4x} - e^{4x} \times 6 \times (2x + 3)^5 \times 2}{[(2x + 3)^6]^2}$  A1  
 $\frac{dy}{dx} = \frac{8xe^{4x}}{(2x + 3)^7}$  (correct numerator) A1  
(correct denominator) A1



7. (a) (i)  $\int e^{5x/6} dx = k \times e^{5x/6} + c$  ( $k = 1, \frac{5}{6}, \frac{6}{5}$ ) M1  
 $\int e^{5x/6} dx = \frac{6}{5} \times e^{5x/6} + c$  A1
- (ii)  $\int (8x + 1)^{1/3} dx = \frac{k \times (8x + 1)^{4/3}}{4/3} + c$  ( $k = 1, 8, \frac{1}{8}$ ) M1  
 $\int (8x + 1)^{1/3} dx = \frac{3}{32} \times (8x + 1)^{4/3} + c$  A1
- (iii)  $\int \sin(1 - x/3) dx = k \times \cos(1 - x/3) + c$  ( $k = -1, 3, -3, \frac{1}{3}$ ) M1  
 $\int \sin(1 - x/3) dx = 3 \times \cos(1 - x/3) + c$  A1

**Note: The omission of the constant of integration is only penalised once.**

- (b)  $\int \frac{1}{4x - 1} dx = k \times \ln(4x - 1)$  ( $k = 1, 4, \frac{1}{4}$ ) M1  
 $\int \frac{1}{4x - 1} dx = \frac{1}{4} \times \ln(4x - 1)$  A1  
 $k \times [\ln(4a - 1) - \ln 7] = 0.284$  ( $k = 1, 4, \frac{1}{4}$ ) m1  
 $\frac{4a - 1}{7} = e^{1.136}$  (o.e.) (c.a.o.) A1  
 $a = 5.7$  (f.t.  $a = 2.6$  for  $k = 1$  and  $a = 2.1$  for  $k = 4$ ) A1

8. Trying to solve  $3x + 4 = 2(x - 3)$  M1  
Trying to solve  $3x + 4 = -2(x - 3)$  M1  
 $x = -10, x = 0.4$  (c.a.o.) A1

**Alternative mark scheme**

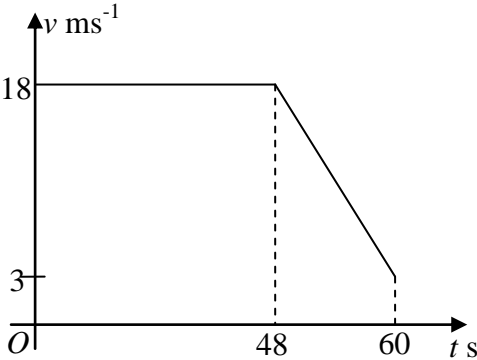
- $(3x + 4)^2 = [2(x - 3)]^2$  (squaring both sides) M1  
 $5x^2 + 48x - 20 = 0$  (at least two coefficients correct) A1  
 $x = -10, x = 0.4$  (c.a.o.) A1

9. (a)  $y - 1 = \frac{2}{\sqrt{3x - 5}}$  B1
- An attempt to isolate  $3x - 5$  by crossmultiplying and squaring M1
- $x = \frac{1}{3} \left[ 5 + \frac{4}{(y - 1)^2} \right]$  (c.a.o.) A1
- $f^{-1}(x) = \frac{1}{3} \left[ 5 + \frac{4}{(x - 1)^2} \right]$
- (f.t. one slip in candidate's expression for  $x$ ) A1
- (b)  $D(f^{-1}) = (1, 1.5]$  B1 B1
10. (a)  $g'(x) = \frac{4}{(x + 1)^2}$  B1
- $g'(x) > 0 \Rightarrow g$  is an increasing function B1
- (b)  $R(g) = (0, 4)$  B1 B1
- (c)  $D(fg) = (-\infty, -2)$  B1
- $R(fg) = (\sqrt{5}, \sqrt{21})$  (f.t. candidate's  $R(g)$ ) B1
- (d) (i)  $fg(x) = \left( \frac{[-4]}{[x + 1]} + 5 \right)^{1/2}$  B1
- (ii) Putting expression for  $fg(x)$  equal to 3 and squaring both sides M1
- $\left[ \frac{-4}{x + 1} \right]^2 = 4$  (o.e.) (c.a.o.) A1
- $x = -3, 1$  (two values, f.t. one slip) A1
- Rejecting  $x = 1$  and thus  $x = -3$  (c.a.o.) A1

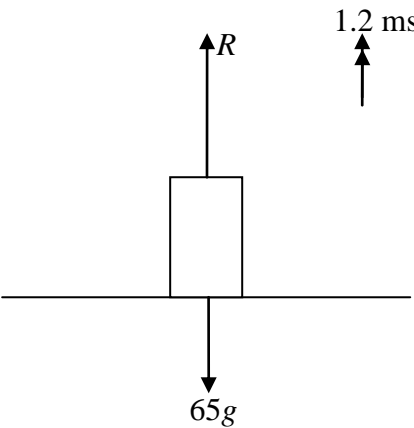
# Mathematics M1 January 2014

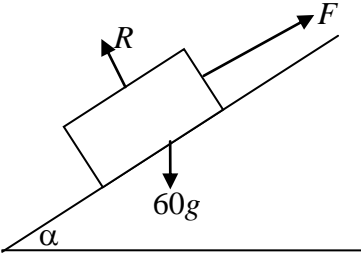
## Solutions and Mark Scheme

### Final Version

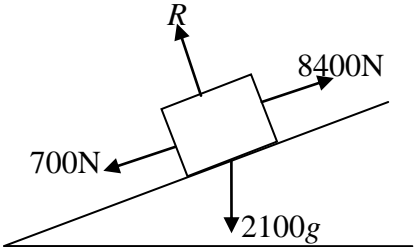
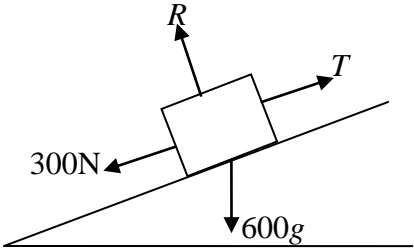
Q	Solution	Mark	Notes
1(a)	 <p>The graph shows velocity <math>v</math> in <math>\text{ms}^{-1}</math> on the vertical axis and time <math>t</math> in seconds on the horizontal axis. The origin is labeled <math>O</math>. The velocity is constant at <math>18 \text{ ms}^{-1}</math> from <math>t = 0</math> to <math>t = 48</math>. From <math>t = 48</math> to <math>t = 60</math>, the velocity decreases linearly to <math>3 \text{ ms}^{-1}</math>. Dashed lines indicate the points <math>(48, 18)</math> and <math>(60, 3)</math> on the graph.</p>	<p>B1 B1</p>	<p><math>(0, 18)</math> to <math>(48, 18)</math> Or <math>(48, 18)</math> to <math>(60, 3)</math> graph all correct, with units, labels.</p>
1(b)	<p>magnitude of deceleration = <math>\frac{18 - 3}{12}</math> = <math>\underline{1.25 \text{ (ms}^{-2}\text{)}}</math></p>	<p>M1 A1</p>	<p>A0 if negative</p>
1(c)	<p>Distance = area under graph Distance = <math>48 \times 18 + 0.5(18 + 3) \times 12</math> Distance = <math>\underline{990 \text{ (m)}}</math></p>	<p>M1 B1 A1</p>	<p>attempt at total area. one correct area seen cao</p>

Q	Solution	Mark	Notes
2(a)	Use of $v = u + at$ , $v=0$ , $u=(\pm)7$ , $a=(\pm)9.8$ $0 = 7 - 9.8t$ $t = \frac{7}{9.8} = \frac{5}{7}(\text{s})$	M1 A1	oe correct equ solvable for $t$ A1
2(b)	Use of $s = ut + 0.5at^2$ , $u=(\pm)7$ , $a=(\pm)9.8$ , $t=4$ $s = 7 \times 4 + 0.5(-9.8) \times 4^2$ $s = 28 - 4.9 \times 16$ $s = -50.4$ Height of cliff is <u>50.4 (m)</u>	M1 A1  A1	if staged method, one correct distance  cao, allow -ve

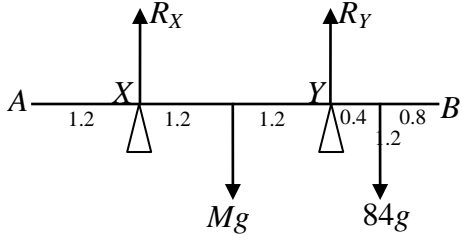
Q	Solution	Mark	Notes
3	 <p>N2L applied to man</p> $R - 65g = 65a$ $R = 65 \times 1.2 + 65 \times 9.8$ $R = \underline{715 \text{ (N)}}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>dim correct and <math>R</math> and <math>65g</math> opposing.</p> <p>cao</p>

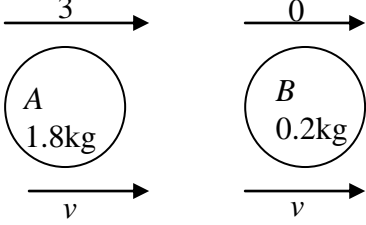
Q	Solution	Mark	Notes
4(a)(i)	 <p> <math>R = 60g \cos \alpha</math>  <math>F = \mu R</math>  <math>F = 60 \times 9.8 \cos \alpha \times 0.3</math>  <math>F = \underline{159.87 \text{ (N)}}</math> </p>	<p>B1</p> <p>B1</p>	
4(a)(ii)	<p>N2L applied to object</p> $60g \sin \alpha - F = 60a$ $60a = 60 \times 9.8 \sin 25^\circ - 159.87$ $a = \underline{1.48 \text{ (ms}^{-2}\text{)}}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>all forces, dim correct.</p> <p>ft <math>F</math></p>
4(b)	<p>If object remains stationary, component Of weight down slope <math>\leq</math> Friction</p> $60g \sin \alpha \leq \mu \times 60g \cos \alpha$ $\therefore \text{least } \mu = \tan 25^\circ$ $= 0.4663$ $= \underline{0.47 \text{ (to 2 d.p.)}}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>si</p>

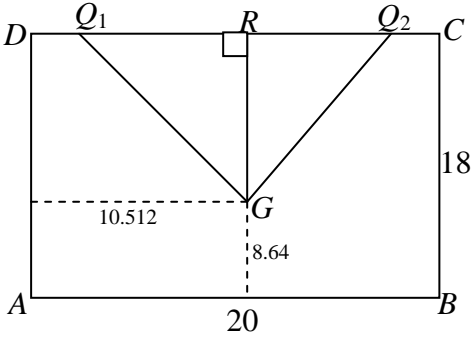
Q	Solution	Mark	Notes
5	<p>Resolve in <math>Q</math> direction</p> $Q = 9\sin 60^\circ$ $= 9 \times \frac{\sqrt{3}}{2} = \underline{7.794}$ <p>Resolve in <math>P</math> direction</p> $P + 9\cos 60^\circ = 6$ $P = 6 - 9 \times 0.5$ $P = \underline{1.5}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>equation required</p> <p>cao</p> <p>equation required, all forces</p> <p>correct equation</p> <p>cao</p>

Q	Solution	Mark	Notes
6(a)	 <p>N2L on whole system</p> $8400 - 700 - 2100g\sin\alpha = 2100a$ $8400 - 700 - 5762.4 = 2100a$ $a = \underline{0.923 \text{ (ms}^{-2}\text{)}}$	<p>M2</p> <p>(M1</p> <p>A2</p> <p>A1</p>	<p>all forces in same dir, dim correct. 8400N and resistance opposing.</p> <p>one force missing but must have comp of wt. and resistance.)</p> <p>-1 each error</p> <p>cao 3 dp required.</p>
6(b)	 <p>N2L applied to trailer</p> $T - 300 - 600g\sin\alpha = 600a$ $T - 300 - 600 \times 9.8 \times \frac{7}{25} = 600 \times \frac{346}{375}$ $T = \underline{2500 \text{ (N)}}$	<p>M1</p> <p>A2</p> <p>A1</p>	<p>all forces, no extra. Dim correct. Either resist. or comp wt opposing</p> <p>-1 each error</p> <p>ft a. answers rounding to 2500</p>



Q	Solution	Mark	Notes
7(a)			
7(a)(i)	Moments about Y  $Mg \times 1.2 = R_X \times 2.4 + 84g \times 0.4$ $(9.8 \times 1.2)M = 2.4 \times 156.8 + 84 \times 9.8 \times 0.4$ $M = \underline{60}$	M1 B1 A1	dim. Correct, all forces, equation, oe any correct moment.
7(a)(ii)	Resolve vertically $R_X + R_Y = Mg + 84g$ $R_Y = 144 \times 9.8 - 156.8$ $R_Y = \underline{1254.4 \text{ (N)}}$	M1 A1 A1	all forces  ft $M$
7(b)(i)	When plank about to tilt about Y $R_Y = 0$ Resolve vertically $R_X = 60g + 84g$ $R_X = \underline{1411.2 \text{ (N)}}$	M1 M1 A1	si all forces ft $M$
7(b)(ii)	Moments about X $84g \times x = 60g \times 1.2$ $x = \frac{6}{7} = \underline{0.86}$ Distance of the person from X = 0.86 (m)	M1 A1	dim correct ft $M$

Q	Solution	Mark	Notes
8(a)(i)	 <p>Conservation of momentum  <math>1.8 \times 3 + 0.2 \times 0 = 1.8v + 0.2v</math>  <math>2v = 5.4</math>  <math>v = \underline{2.7 \text{ (ms}^{-1}\text{)}}</math></p>	M1 A1 A1	allow different $v$ 's  convincing
8(a)(ii)	$e = \underline{0}$	B1	
8(b)(i)	N2L applied to combined object $-8 = 2a$ $a = -4 \text{ ms}^{-2}$ $ a  = \underline{4 \text{ (ms}^{-2}\text{)}}$	M1 A1	dim correct
8(b)(ii)	Use of $v = u + at$ , $u = 2.7$ , $a = (\pm)4$ , $t = 0.5$ $v = 2.7 - 4 \times 0.5$ $v = \underline{0.7 \text{ (ms}^{-1}\text{)}}$	M1 A1 A1	oe ft $a$ if $<0$ . ft $a$ if $<0$ .
8(b)(iii)	Use of $v^2 = u^2 + 2as$ , $u = 2.7$ , $v = 2$ , $a = (\pm)4$ $2^2 = 2.7^2 - 2 \times 4s$ $s = \underline{0.41(125 \text{ m})}$	M1 A1 A1	oe ft $a$ if $<0$ . ft $a$ if $<0$ .

Q	Solution	Mark	Notes																				
9(a)	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 20%;"></th> <th style="width: 15%;">Area</th> <th style="width: 15%;">from AD</th> <th style="width: 15%;">from AB</th> </tr> </thead> <tbody> <tr> <td>ABCD</td> <td>360</td> <td>10</td> <td>9</td> </tr> <tr> <td>Circle</td> <td>21</td> <td>6</td> <td>12</td> </tr> <tr> <td>XYZ</td> <td>36</td> <td>13</td> <td>7</td> </tr> <tr> <td>Lamina</td> <td>375</td> <td><math>x</math></td> <td><math>y</math></td> </tr> </tbody> </table>		Area	from AD	from AB	ABCD	360	10	9	Circle	21	6	12	XYZ	36	13	7	Lamina	375	$x$	$y$	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>all 4 correct areas</p>
	Area	from AD	from AB																				
ABCD	360	10	9																				
Circle	21	6	12																				
XYZ	36	13	7																				
Lamina	375	$x$	$y$																				
9(a)(i)	<p>Moments about AD</p> $360 \times 10 + 36 \times 13 = 375x + 21 \times 6$ $x = \underline{10.5(12 \text{ cm})}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>consistent use of signs for areas and moments.</p> <p>ft table if +XYZ and -circ</p> <p>cao</p>																				
9(a)(ii)	<p>Moments about AB</p> $360 \times 9 + 36 \times 7 = 375y + 21 \times 12$ $y = \underline{8.6(4 \text{ cm})}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>consistent use of signs for areas and moments.</p> <p>ft table if +XYZ and -circ</p> <p>cao</p>																				
9(b)	 <p>Consider triangle <math>RQ_1G</math>  Angle <math>RGQ = \text{angle } RQG = 45^\circ</math>  <math>\therefore RQ = RG</math></p> <p>Let <math>DQ_1 = x</math>  <math>10.512 - x = 18 - 8.64</math>  <math>x = 10.512 - 9.36</math>  <math>DQ_1 = \underline{1.1(52 \text{ cm})}</math></p> <p><math>DQ_2 = 10.512 + (18 - 8.64)</math>  <math>DQ_2 = \underline{19.8(72 \text{ cm})}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>ft <math>x, y</math></p> <p>ft <math>x, y</math></p>																				



Ques	Solution	Mark	Notes
4(a)(i) (ii) (b)	$P(X = 6) = \binom{20}{6} \times 0.2^6 \times 0.8^{14} = 0.109$ Prob = 0.9900 – 0.0692 or 0.9308 – 0.0100 = 0.921 cao  B(200,0.0123) is approx Po(2.46) $P(Y = 3) = \frac{e^{-2.46} \times 2.46^3}{3!} = 0.212$	M1A1  B1B1 B1  B1  M1A1	M0 if no working shown  B0B0B0 if no working shown    M0 if no working shown Do not accept use of tables
5(a) (b)	$P(2G) = \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{4} \times \frac{1}{3}$ $= \frac{5}{9} \text{ cao}$ $P(A 2G) = \frac{1/3}{5/9}$ $= \frac{3}{5} \text{ cao}$	M1A3  A1  B1B1  B1	M1 Use of Law of Total Prob (Accept tree diagram)    FT denominator from (a) B1 num, B1 denom
6(a)(i) (ii) (b)(i) (ii)	X is B(10,0.75) si $E(X) = 7.5,$ $\text{Var}(X) = 1.875$  Attempt to evaluate either $P(X = 7)$ or $P(X = 8)$ $P(X = 7) = 0.250$ ; $P(X = 8) = 0.282$ So try $P(X = 9) = 0.188$ Most likely value = 8  $W = 10X - 2(10 - X) = 12X - 20$ $E(W) = 12 \times 7.5 - 20 = 70$ $\text{Var}(W) = 12^2 \times \text{Var}(X) = 270$	B1 B1 B1  M1 A1 A1 A1  B1 B1 M1A1	Award the final A1 only if the previous A1 was awarded  FT their mean and variance from (a) and FT their derived values of $a$ and $b$ provided that $a \neq 1$ and $b \neq 0$
7(a) (b)(i) (ii)	$E(X) = 0.1 \times 1 + 0.2 \times 2 + 0.3 \times 3 + 0.1 \times 4 + 0.3 \times 5$ $= 3.3$ $E(X^2) = 0.1 \times 1 + 0.2 \times 4 + 0.3 \times 9 + 0.1 \times 16$ $+ 0.3 \times 25 \quad (12.7)$ $\text{Var}(X) = 12.7 - 3.3^2 = 1.81$  The possibilities are (1,1,2); (1,2,1); (2,1,1) $P(S = 4) = 0.1^2 \times 0.2 \times 3 = 0.006$  The only extra possibility is (1,1,1) so $P(S = 3) = 0.1^3 \quad (0.001)$ Therefore $P(S \leq 4) = 0.007$	M1  A1 B1  M1A1  B1 M1A1  B1 B1 B1	FT their $E(X^2)$    Award M1 if only one correct possibility given  FT from (b)(i) if M1 awarded

Ques	Solution	Mark	Notes
8(a)(i)	$\text{Prob} = \frac{e^{-15} \times 15^{12}}{12!} \quad \text{or } 0.2676 - 0.1848$ $= 0.083 \quad \text{or } 0.8152 - 0.7324$	<b>M1</b> <b>A1</b>	M0 if no working shown
(ii)	We require $P(X \geq 20)$ $= 1 - 0.8752 = 0.1248$	<b>M1</b> <b>A1</b>	Award M1A0 for use of adjacent row or column
(b)	(Using tables, the number required is) 25	<b>M1A1</b>	Award M1A0 for 24 or 26
9(a)(i)	Using $F(2) = 1$ $1 = k(8 - 2)$ $k = 1/6 \text{ (convincing)}$	<b>M1</b> <b>A1</b>	
(ii)	$P(1.25 \leq X \leq 1.75) = F(1.75) - F(1.25)$ $= 0.6015\dots - 0.1171\dots \text{ si}$ $= 0.484 \text{ (31/64)}$	<b>M1</b> <b>A1</b> <b>A1</b>	
(b)(i)	$f(x) = \frac{d}{dx} \left( \frac{x^3 - x}{6} \right)$ $= \frac{3x^2 - 1}{6}$	<b>M1</b> <b>A1</b>	
(ii)	$E(X) = \int_1^2 x \left( \frac{3x^2 - 1}{6} \right) dx$ $= \left[ \frac{x^4}{8} - \frac{x^2}{12} \right]_1^2$ $= 1.625 \quad \text{cao}$	<b>M1A1</b> <b>A1</b> <b>A1</b>	M1 for the integral of $xf(x)$ , A1 for completely correct with or without limits FT on their $f$ if previous M1 awarded Limits must appear here if not before M0 if no working shown

# Mathematics FP1 January 2014

## Solutions and Mark Scheme

### Final Version

Ques	Solution	Mark	Notes
<b>1</b>	$f(x+h) - f(x) = \frac{x+h}{1+x+h} - \frac{x}{1+x}$ $= \frac{(x+h)(1+x) - x(1+x+h)}{(1+x+h)(1+x)}$ $= \frac{h}{(1+x+h)(1+x)}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h}{h(1+x+h)(1+x)}$ $= \frac{1}{(1+x)^2} \text{ CSO}$	<b>M1A1</b>  <b>A1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	
<b>2</b>	$S_n = \sum_{r=1}^n r(r+1)^2 = \sum_{r=1}^n r^3 + 2\sum_{r=1}^n r^2 + \sum_{r=1}^n r$ $= \frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$ $= \frac{n(n+1)}{12} (3n^2 + 3n + 8n + 4 + 6)$ $= \frac{n(n+1)(n+2)(3n+5)}{12}$	<b>M1A1</b>  <b>A1A1</b>  <b>A1</b>  <b>A1</b>	Award A1 for 2 correct
<b>3(a)</b>	$(1+2i)^4 = 1 + 4.2i + 6(2i)^2 + 4(2i)^3 + (2i)^4$ $= 1 + 8i - 24 - 32i + 16 = -7 - 24i$	<b>M1</b> <b>A1</b>	Award M1 for use of binomial theorem (oe)
<b>(b)(i)</b>	<p>Let <math>f(x) = x^4 + 12x - 5</math></p> $f(1+2i) = -7 - 24i + 12 + 24i - 5 = 0$ <p>(showing that <math>1+2i</math> is a root)</p>	<b>M1A1</b>	
<b>(ii)</b>	<p>Another root is <math>1-2i</math></p> <p>EITHER</p> <p>It follows that <math>x^2 - 2x + 5</math> is a factor of <math>f(x)</math></p> $x^4 + 12x - 5 = (x^2 - 2x + 5)(x^2 + 2x - 1)$ <p>The other two roots are <math>-1 \pm \sqrt{2}</math></p> <p>OR</p> $(1+2i)(1-2i) = 5$ $(1+2i) + (1-2i) = 2$ <p>Therefore if <math>\alpha, \beta</math> denote the other two roots</p> $\alpha + \beta = -2 \text{ and } \alpha\beta = -1$ <p>So <math>\alpha, \beta</math> are the roots of the equation <math>x^2 + 2x - 1 = 0</math></p> <p>The other two roots are <math>-1 \pm \sqrt{2}</math></p>	<b>B1</b>  <b>B1</b>  <b>M1A1</b>  <b>M1A1</b>  <b>B1</b>  <b>B1</b>  <b>M1A1</b>	

Ques	Solution	Mark	Notes
4	$\alpha + \beta = \frac{3}{2}, \alpha\beta = 2$ $\alpha^2\beta + \alpha\beta^2 + \alpha\beta = \alpha\beta(\alpha + \beta + 1) = 5$ $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2 = \alpha^2\beta^2(\alpha\beta + \alpha + \beta) = 14$ $\alpha\beta^2 \times \alpha^2\beta \times \alpha\beta = \alpha^4\beta^4 = 16$ <p>The required equation is</p> $x^3 - 5x^2 + 14x - 16 = 0 \text{ cao}$	<p><b>B1</b></p> <p><b>M1A1</b></p> <p><b>M1A1</b></p> <p><b>M1A1</b></p> <p><b>B1</b></p>	<p>FT one slip in line above in sign or in their two values.</p> <p>FT their three values</p>
5(a)	<p>Ref matrix = <math>\begin{bmatrix} 0 &amp; -1 &amp; 0 \\ -1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>Translation matrix = <math>\begin{bmatrix} 1 &amp; 0 &amp; 1 \\ 0 &amp; 1 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>Rotation matrix = <math>\begin{bmatrix} 0 &amp; 1 &amp; 0 \\ -1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> $\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>m1</b></p> <p><b>A1</b></p> <p><b>M1A1</b></p>	
(b)	<p>The general point on the line is <math>(\alpha, 2\alpha - 1)</math>.</p> <p>Consider</p> $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ 2\alpha - 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\alpha + 2 \\ 2\alpha - 2 \\ 1 \end{bmatrix}$ $x = -\alpha + 2, y = 2\alpha - 2$ <p>Eliminating <math>\alpha</math>, the equation of the image is <math>y = 2 - 2x</math></p>		



Ques	Solution	Mark	Notes
<b>6(a)</b>	Putting $n = 1$ , the formula gives $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ which is correct so the result is true for $n = 1$ Assume formula is true for $n = k$ , ie $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^k = \begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix}$ Consider, for $n = k + 1$ , $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^k \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^k$ $= \begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix}$ $= \begin{bmatrix} 1 & 2 + 3(3^k - 1) \\ 0 & 3^{k+1} \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 3^k - 1 + 2 \cdot 3^k \\ 0 & 3^{k+1} \end{bmatrix}$ $= \begin{bmatrix} 1 & 3^{k+1} - 1 \\ 0 & 3^{k+1} \end{bmatrix}$ Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$ , the result is proved by induction.	<b>B1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>  <b>A1</b>  <b>A1</b>	       This line must be seen  Award this A1 only if previous A1 awarded  Award final A1 only if all six previous marks have been awarded
<b>(b)</b>	The formula gives $\mathbf{A}^{-1} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}$ EITHER Consider $\begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR $\mathbf{A}^{-1} = \frac{1}{3} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}$ The formula is therefore correct for $n = -1$	<b>B1</b>  <b>B1</b>  <b>B1</b>	

Ques	Solution	Mark	Notes
<p><b>7(a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)</b></p>	<p>Cofactor matrix = <math>\begin{bmatrix} -1 &amp; 2 &amp; -1 \\ -9 &amp; 6 &amp; 0 \\ 7 &amp; -5 &amp; 1 \end{bmatrix}</math> si</p> <p>Adjugate matrix = <math>\begin{bmatrix} -1 &amp; -9 &amp; 7 \\ 2 &amp; 6 &amp; -5 \\ -1 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>Determinant = 3</p> <p>Inverse matrix = <math>\frac{1}{3} \begin{bmatrix} -1 &amp; -9 &amp; 7 \\ 2 &amp; 6 &amp; -5 \\ -1 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p><math>\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -1 &amp; -9 &amp; 7 \\ 2 &amp; 6 &amp; -5 \\ -1 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 19 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}</math></p>	<p><b>M1</b> <b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Award the M1 if at least 5 of the elements are correct</p> <p>FT their adjugate matrix</p> <p>FT their inverse matrix</p>
<p><b>8(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	<p>Taking logs,</p> <p><math>\ln f(x) = \sqrt{x} \ln\left(\frac{1}{x}\right)</math></p> <p>Differentiating,</p> <p><math>\frac{f'(x)}{f(x)} = \frac{1}{2\sqrt{x}} \ln\left(\frac{1}{x}\right) + \sqrt{x} \cdot -\frac{1}{x}</math></p> <p><math>f'(x) = f(x) \left( \frac{-2 - \ln x}{2\sqrt{x}} \right)</math></p> <p>Putting <math>f'(x) = 0</math>,</p> <p><math>\ln(x) = -2</math> so <math>x = e^{-2} = 0.135</math></p> <p><math>y = e^{2/e} = 2.09</math></p> <p><math>f'(x) &gt; 0</math> for <math>0 &lt; x &lt; e^{-2}</math> ; <math>f'(x) &lt; 0</math> for <math>x &gt; e^{-2}</math> cao</p> <p>It is a maximum</p>	<p><b>B1</b></p> <p><b>B1B1</b></p> <p><b>B1</b></p> <p><b>M1</b> <b>A1</b> <b>A1</b> <b>B1</b></p> <p><b>B1</b></p>	<p>B1 for each side</p> <p>Award this B1 only if <math>\ln(1/x)</math> has been simplified to <math>-\ln x</math> and the two terms are over a common denom.</p> <p>Accept <math>x &lt; e^{-2}</math></p> <p>Award this B1 if the answer is consistent with a previous line containing two sets of values of <math>x</math> even if incorrect.</p>

Ques	Solution	Mark	Notes
<b>9(a)</b>	Putting $z = 0$ , we see that LHS = RHS = 2 hence locus passes through (0,0)	<b>M1</b> <b>A1</b>	Accept alternative arguments that do not depend upon the result obtained in (b)
<b>(b)</b>	Putting $z = x + iy$ , $ x - 2 + iy  = 2 x + i(y + 1) $ $(x - 2)^2 + y^2 = 4(x^2 + (y + 1)^2)$ $x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2 + 8y + 4$ $3x^2 + 3y^2 + 4x + 8y = 0$ (This shows that the locus of P is a circle.) Consider the equation in the form $x^2 + y^2 + \frac{4}{3}x + \frac{8}{3}y = 0$ The centre is $\left(-\frac{2}{3}, -\frac{4}{3}\right)$ cao The radius is $\frac{2\sqrt{5}}{3}$ (1.49) cao	<b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b> <b>B1</b> <b>B1</b> <b>B1</b>	



WJEC  
245 Western Avenue  
Cardiff CF5 2YX  
Tel No 029 2026 5000  
Fax 029 2057 5994  
E-mail: [exams@wjec.co.uk](mailto:exams@wjec.co.uk)  
website: [www.wjec.co.uk](http://www.wjec.co.uk)